TORSTEN HOEFLER

# How fast will your application go? Static and dynamic techniques for application performance modeling

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presented at Oak Ridge National Laboratory, June 2015

# Performance modeling

- **What is this all about???**
  - A wide-spread practitioner's view on performance modeling:


*(replace "meeting" with performance optimization and "premeeting" with performance modeling)*


IBM

# Analytical application performance modeling

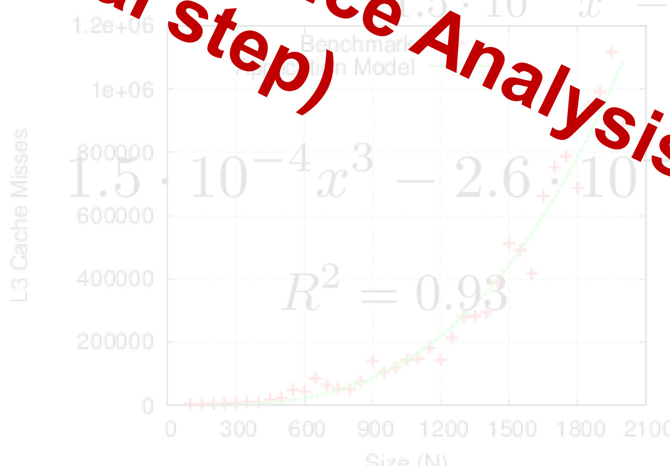
- **Scalability bug prediction**
  - Find latent scalability bugs early on (before machine deployment)
  
    *SC13: A. Calotoiu, TH, M. Poke, F. Wolf: Using Automated Performance Modeling to Find Scalability Bugs in Complex Codes*
- **Automated performance testing**
  - Performance modeling as part of a software engineering discipline in HPC
  
    *ICS'15: S. Shudler, A. Calotoiu, TH, A. Strube, F. Wolf: Exascaling Your Library: Will Your Implementation Meet Your Expectations?*
- **Hardware/Software co-design**
  - Decide how to architect systems
- **Making performance development intuitive**

**Performance Modeling = Performance Analysis v 2.0**
**(the next logical step)**

$$1.5 \cdot 10^{-4} x^3 - 2.6 \cdot 10^{-2} x^2$$
$$R^2 = 0.93$$




vs.



# Manual analytical performance modeling

**Identify kernels**
- Parts of the program that dominate its performance at larger scales
- Identified via small-scale tests and intuition

**Create models**
- Laborious process
- Still confined to a small community of skilled experts

- **Disadvantages**
  - Time consuming
  - Error-prone, may overlook unscalable code

TH, W. Gropp, M. Snir, and W. Kramer: Performance Modeling for Systematic Performance Tuning, SC11

# Our first step: scalability bug detector

```
main() {
  foo()
  bar()
  compute()
}
```

Instrumentation

- All functions

Performance measurements (profiles)

$p_1 = 128$ | $p_2 = 256$ | $p_3 = 512$ | $p_4 = 1,024$ | $p_5 = 2,048$ | $p_6 = 4,096$

Weak scaling

**Input**

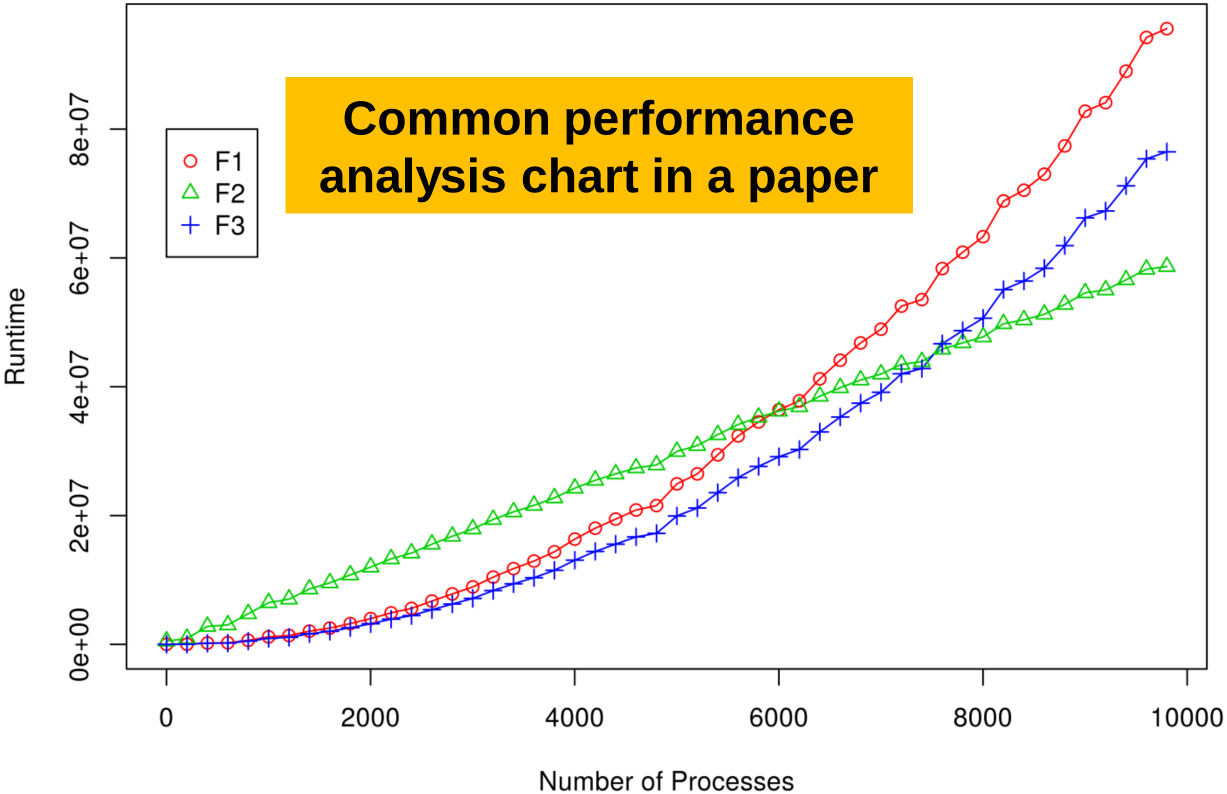
Automated modeling

**Output**

Ranking:
1. Asymptotic
2. Target scale $p_t$

1. foo
2. compute
3. main
4. bar
[…]

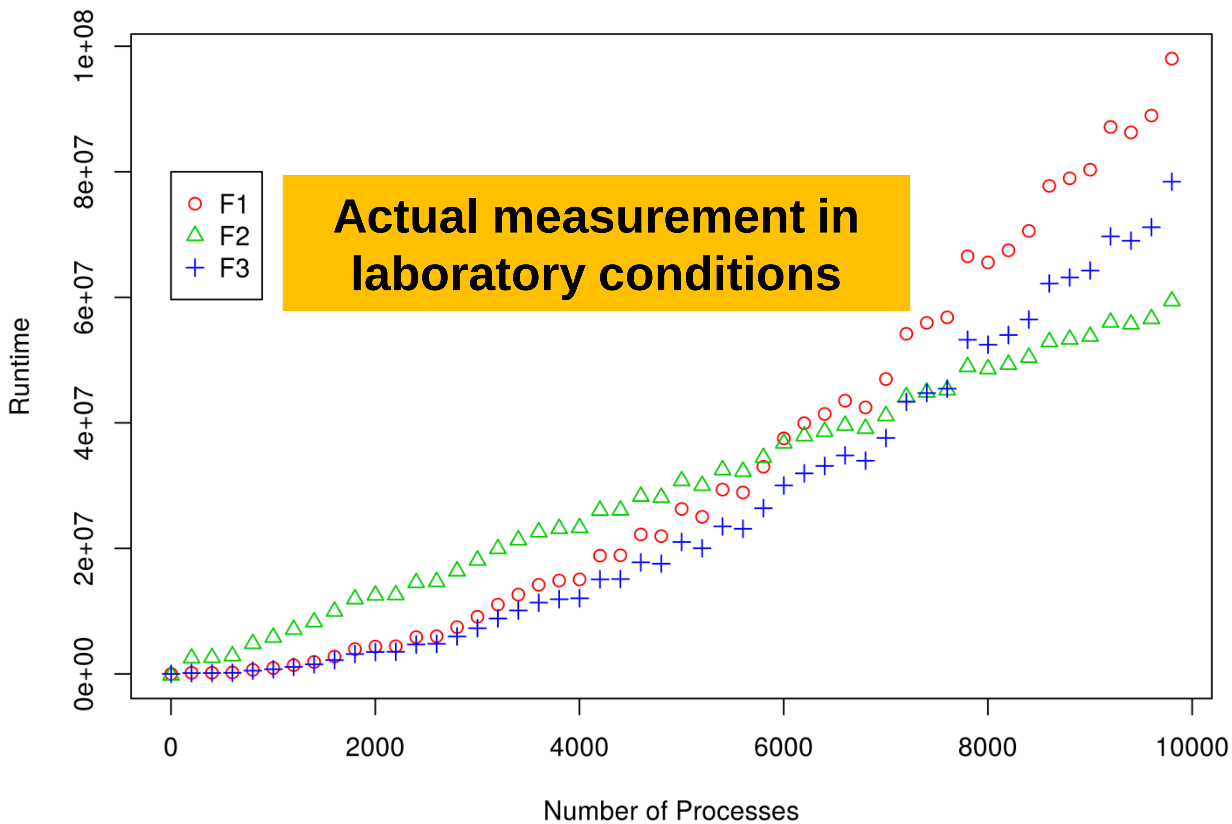
# Primary focus on scaling trend




## Our ranking

1. $F_1$
2. $F_3$
3. $F_2$

# Primary focus on scaling trend




## Our ranking

1. $F_1$
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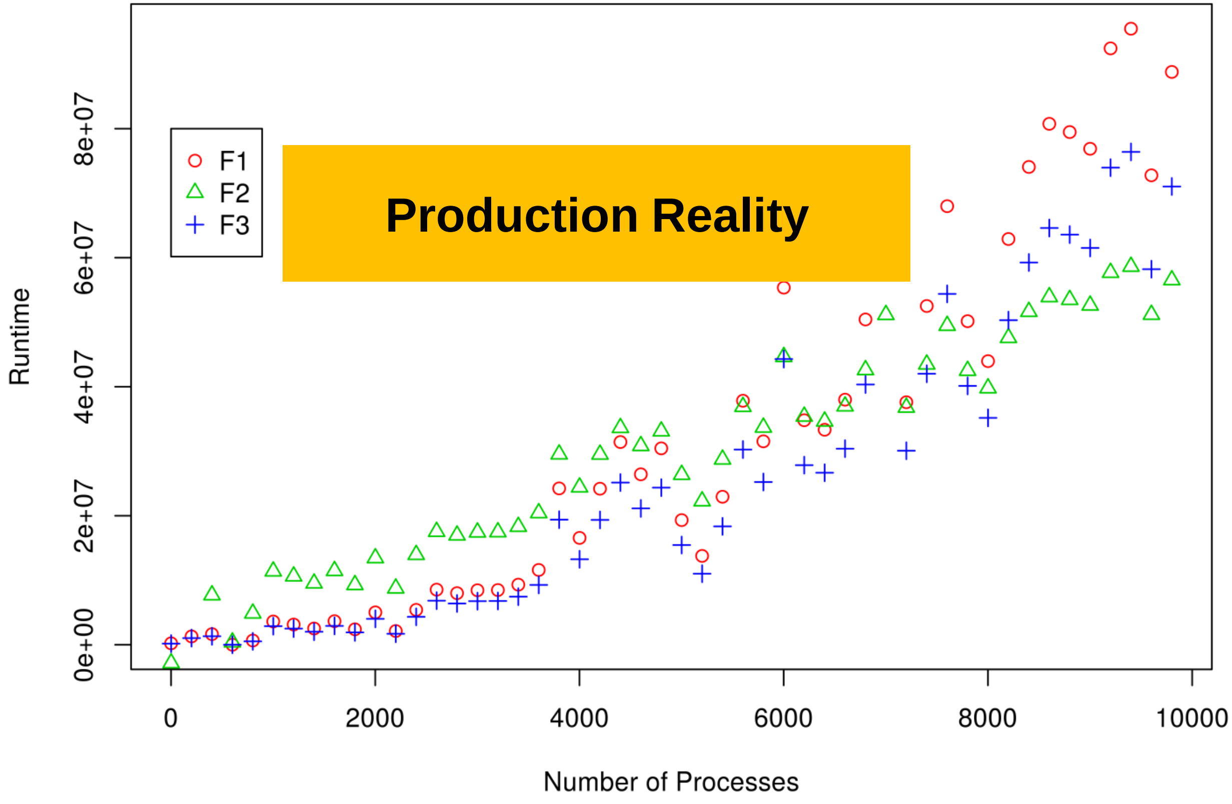
# Primary focus on scaling trend




## Our ranking

1. $F_1$
2. $F_3$
3. $F_2$

# How to mechanize the expert? → Survey!

**Computation**

LU
$t(p) \sim c$

FFT
$t(p) \sim \log_2(p)$

Naïve N-body
$t(p) \sim p$

...

Samplesort
$t(p) \sim p^2 \log_2^2(p)$

**Communication**

LU
$t(p) \sim c$

FFT
$t(p) \sim \log_2(p)$

Naïve N-body
$t(p) \sim p$

...

Samplesort
$t(p) \sim p^2$

# Survey result: performance model normal form

$$f(p) = \sum_{k=1}^{n} c_k \cdot p^{i_k} \cdot \log_2^{j_k}(p)$$

$n \in \mathbb{N}$
$i_k \in I$
$j_k \in J$
$I, J \subset \mathbb{Q}$

$n = 1$

$I = \{0, 1, 2\}$

$J = \{0, 1\}$

$c_1$ | $c_1 \cdot \log(p)$
$c_1 \cdot p$ | $c_1 \cdot p \cdot \log(p)$
$c_1 \cdot p^2$ | $c_1 \cdot p^2 \cdot \log(p)$

# Survey result: performance model normal form

$$f(p) = \sum_{k=1}^{n} c_k \cdot p^{i_k} \cdot \log_2^{j_k}(p)$$

$n \in \mathbb{N}$

$i_k \in I$

$n = 2$

$I = \{0, 1, 2\}$

$J = \{0, 1\}$

$c_1 + c_2 \cdot p$

$c_1 + c_2 \cdot p^2$

$c_1 + c_2 \cdot \log(p)$

$c_1 + c_2 \cdot p \cdot \log(p)$

$c_1 + c_2 \cdot p^2 \cdot \log(p)$

$c_1 \cdot \log(p) + c_2 \cdot p$

$c_1 \cdot \log(p) + c_2 \cdot p \cdot \log(p)$

$c_1 \cdot \log(p) + c_2 \cdot p^2$

$c_1 \cdot \log(p) + c_2 \cdot p^2 \cdot \log(p)$

$c_1 \cdot p + c_2 \cdot p \cdot \log(p)$

$c_1 \cdot p + c_2 \cdot p^2$

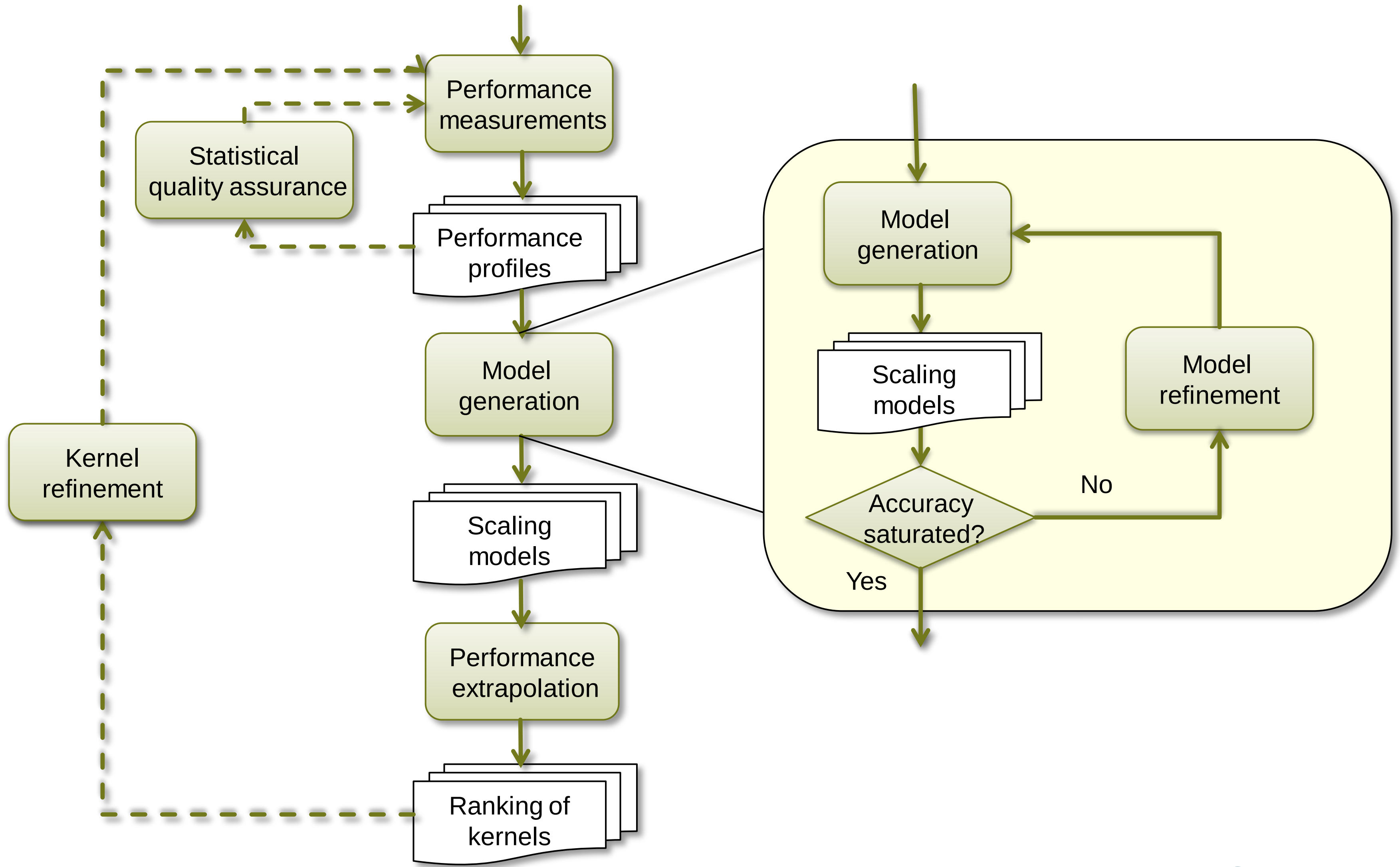
$c_1 \cdot p + c_2 \cdot p^2 \cdot \log(p)$

$c_1 \cdot p \cdot \log(p) + c_2 \cdot p^2$

$c_1 \cdot p \cdot \log(p) + c_2 \cdot p^2 \cdot \log(p)$

$c_1 \cdot p^2 + c_2 \cdot p^2 \cdot \log(p)$

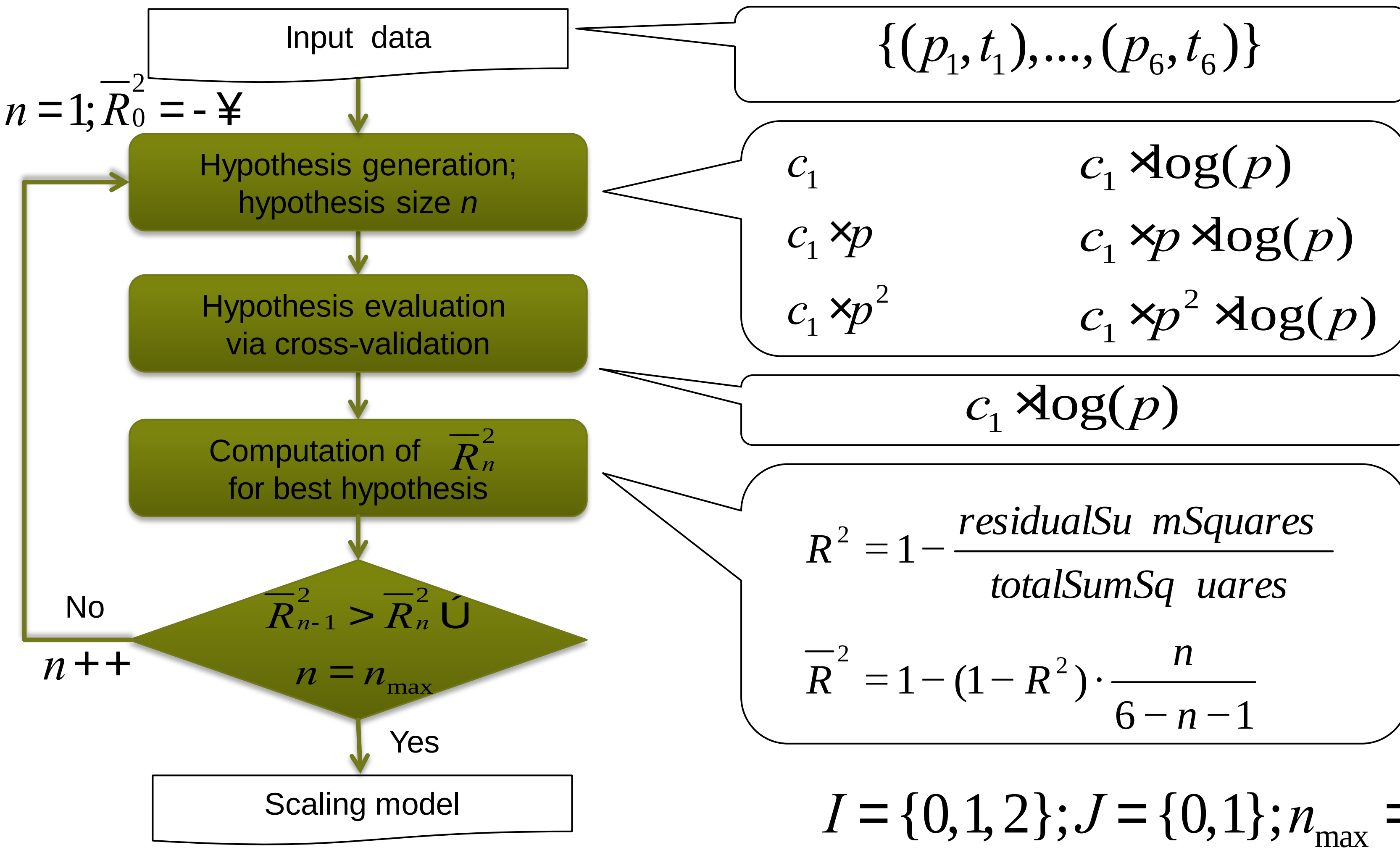
# Our automated generation workflow




scalasca

A. Calotoiu, T. Hoefler, M. Poke, F. Wolf: Using Automated Performance Modeling to Find Scalability Bugs in Complex Codes, SC13

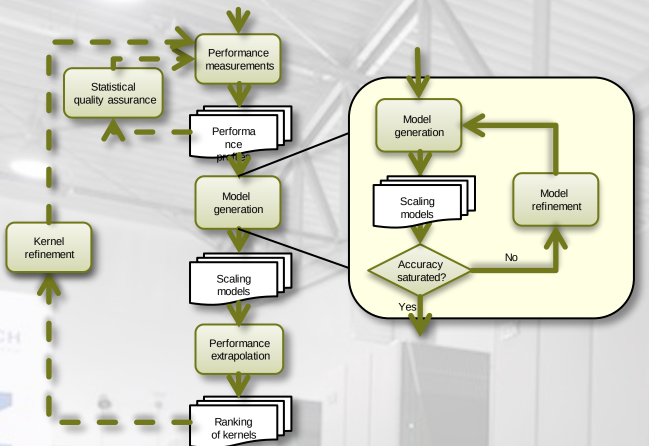
# Model refinement




$$I = \{0, 1, 2\}; J = \{0, 1\}; n_{max} = 2$$


JÜLICH
JUQUEEN
IBM
supercomputer Blue Gene Q

# Evaluation overview

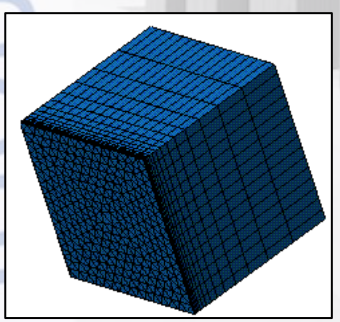



$$I = \left\{\tfrac{0}{2}, \tfrac{1}{2}, \tfrac{2}{2}, \tfrac{3}{2}, \tfrac{4}{2}, \tfrac{5}{2}, \tfrac{6}{2}\right\}$$

$$J = \{0, 1, 2\}$$

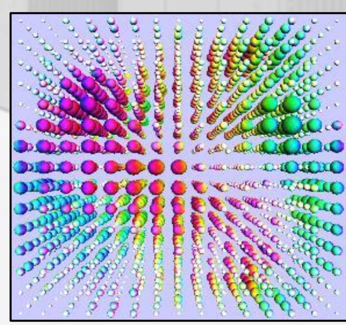
$$n = 5$$

Sweep3D



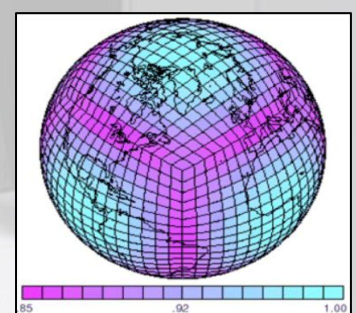
 ✔

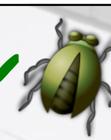
MILC



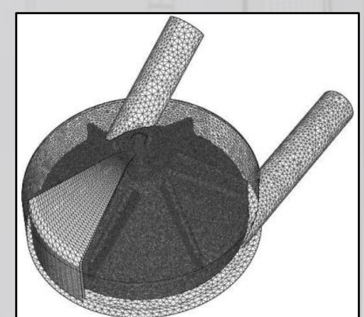
✖

HOMME




 ✔  ✔

XNS

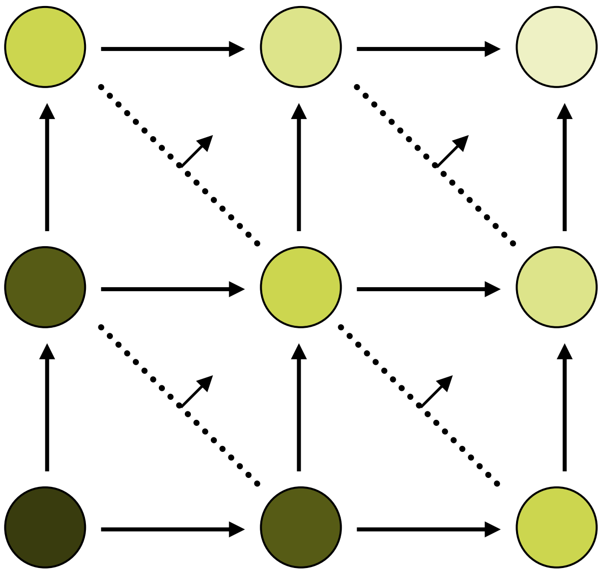


 ✔

# Sweep3D communication performance

- **Solves neutron transport problem**
- **3D domain mapped onto 2D process grid**
- **Parallelism achieved through pipelined wave-front process**

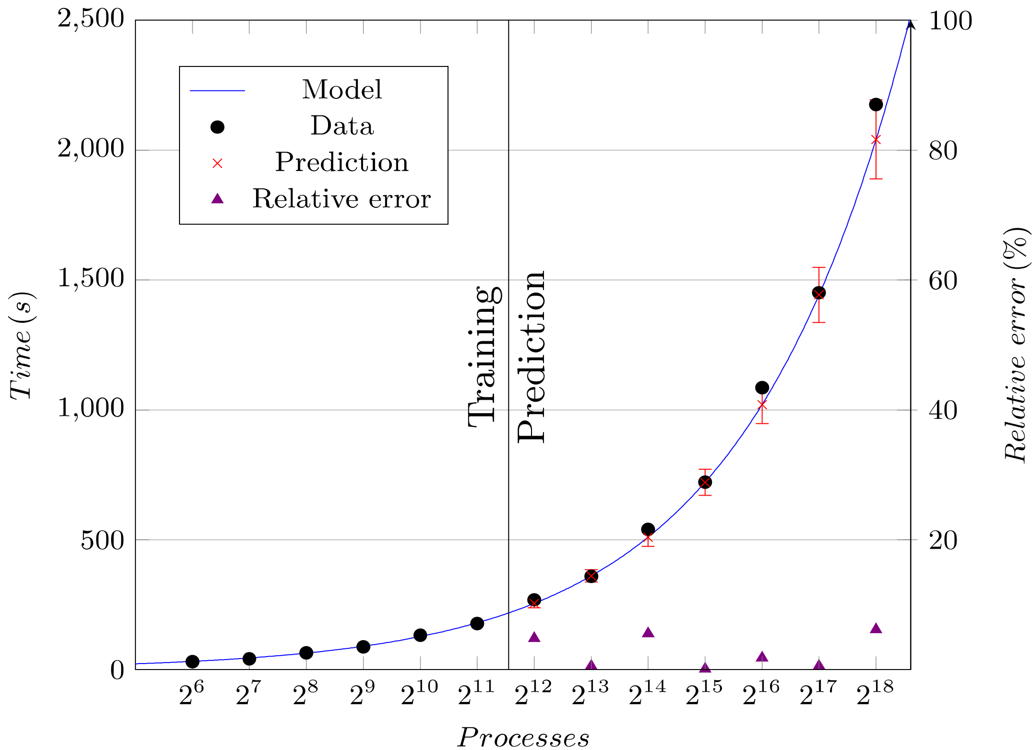
$$t^{comm} = c \cdot \sqrt{p}$$



- **LogGP model for communication developed by Hoisie et al.**
  - We assume $p=p_x*p_y$ → Equation (6) in [1]

[1] A. Hoisie, O. M. Lubeck, and H. J. Wasserman. Performance analysis of wavefront algorithms on very-large scale distributed systems. In Workshop on Wide Area Networks and High Performance Computing, pages 171–187. Springer-Verlag, 1999.

# Sweep3D communication performance

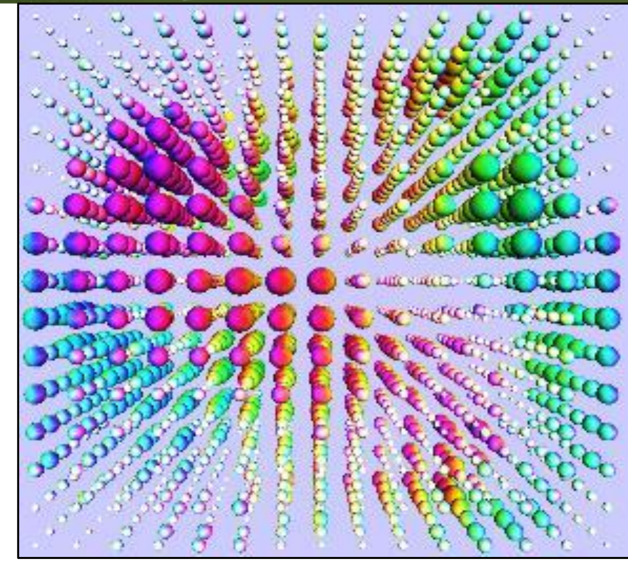



| Kernel [2 of 40] | Runtime[%] $p_t$=262k | Model [s] t = f(p) | Predictive error [%] $p_t$=262k |
|---|---|---|---|
| sweep → MPI_Recv | 65.35 | $4.03\sqrt{p}$ | 5.10 |
| sweep | 20.87 | 582.19 | .01 |

#bytes = const.
#msg = const.

$p_i \leq 8k$

# MILC



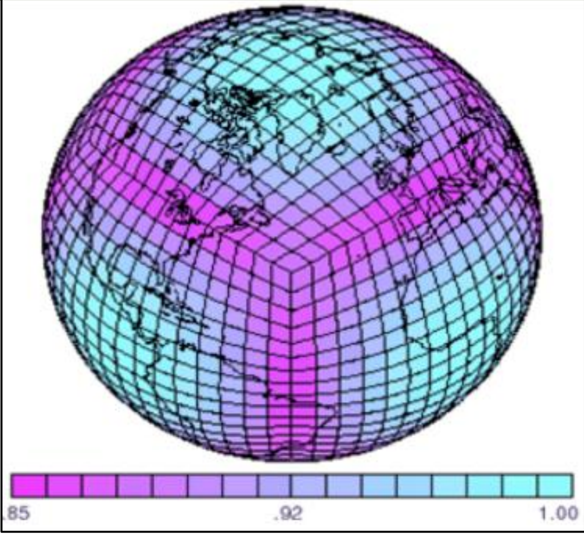
- **MILC/su3_rmd – from MILC suite of QCD codes with performance model manually created**

- **Time per process should remain constant except for a rather small logarithmic term caused by global convergence checks**

| Kernel [3 of 479] | Model [s] t=f(p) | Predictive Error [%] $p_t$=64k |
|---|---|---|
| compute_gen_staple_field | $2.40 \times 10^{-2}$ | 0.43 |
| g_vecdoublesum → MPI_Allreduce | $6.30 \times 10^{-6} \times \log_2^2(p)$ | 0.01 |
| mult_adj_su3_fieldlink_lathwec | $3.80 \times 10^{-3}$ | 0.04 |

$$p_i \leq 16k$$

# HOMME



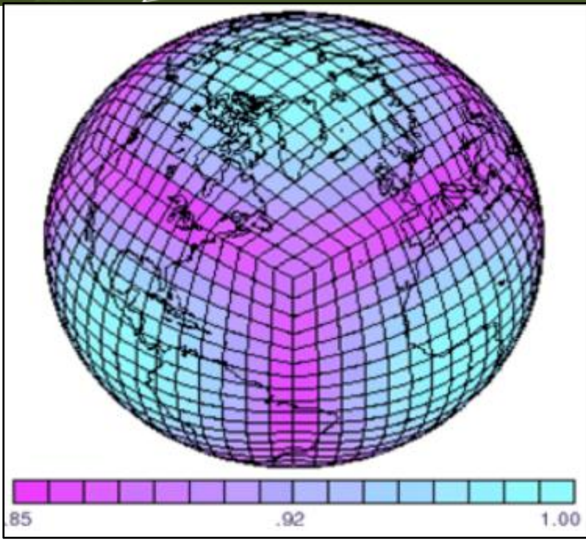

- **Core of the Community Atmospheric Model (CAM)**
- Spectral element dynamical core on a cubed sphere grid

| Kernel [3 of 194] | Model [s] t = f(p) | Predictive error [%] $p_t$ = 130k |
|---|---|---|
| box_rearrange → MPI_Reduce | $0.026 + 2.53 \times 10^{-6} p \times \sqrt{p} + 1.24 \times 10^{-12} p^3$ | 57.02 |
| vlaplace_sphere_vk | 49.53 | 99.32 |
| compute_and_apply_rhs | 48.68 | 1.65 |

$$p_i \leq 15\text{k}$$

# HOMME (2)

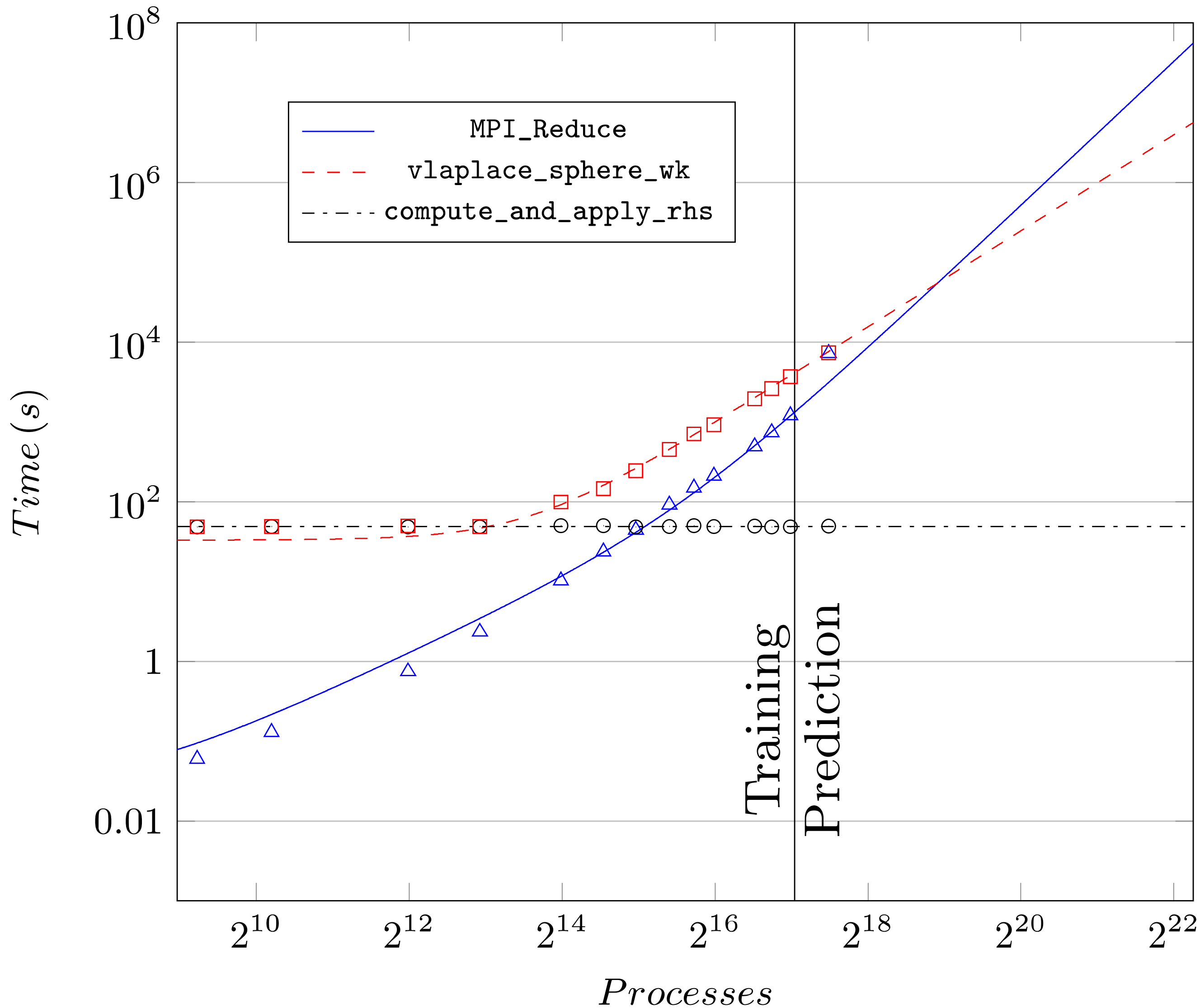
- **Core of the Community Atmospheric Model (CAM)**
- Spectral element dynamical core on a cubed sphere grid




| Kernel [3 of 194] | Model [s] t = f(p) | Predictive error [%] $p_t$ = 130k |
|---|---|---|
| box_rearrange → MPI_Reduce | $3.63\times10^{-6}\,p\times\sqrt{p}+7.21\times10^{-13}\,p^{3}$ | 30.34 |
| vlaplace_sphere_vk | $24.44+2.26\times10^{-7}\,p^{2}$ | 4.28 |
| compute_and_apply_rhs | 49.09 | 0.83 |

$p_i$ £ 43k

# HOMME (3)


$10^8$
$10^6$
$10^4$
$10^2$
1
0.01
Time (s)
MPI_Reduce
vlaplace_sphere_wk
compute_and_apply_rhs
Training
Prediction
$2^{10}$
$2^{12}$
$2^{14}$
$2^{16}$
$2^{18}$
$2^{20}$
$2^{22}$
Processes

# What about strong scaling?

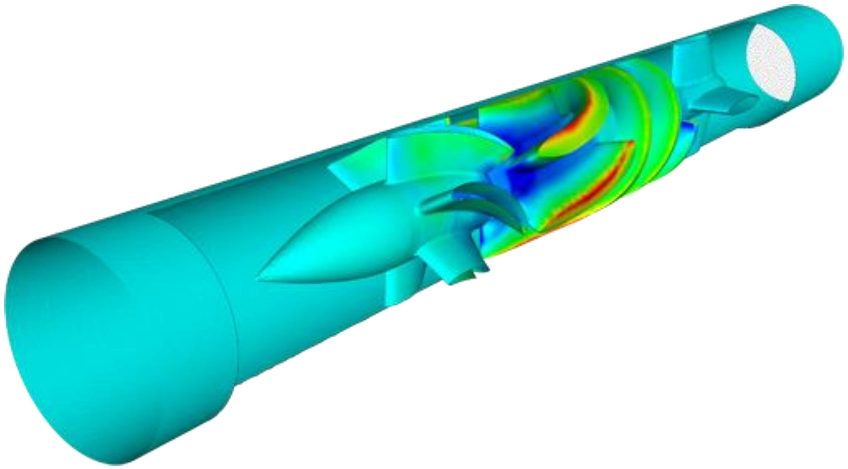
- **Wall-clock time not necessarily monotonically increasing – harder to capture model automatically**
  - Different invariants require different reductions across processes

| | Weak scaling | Strong scaling |
|---|---|---|
| Invariant | Problem size per process | Overall problem size |
| Model target | Wall-clock time | Accumulated time |
| Reduction | Maximum / average | Sum |

- **Superlinear speedup through cache effects**
  - Measure and model re-use distance?

# XNS

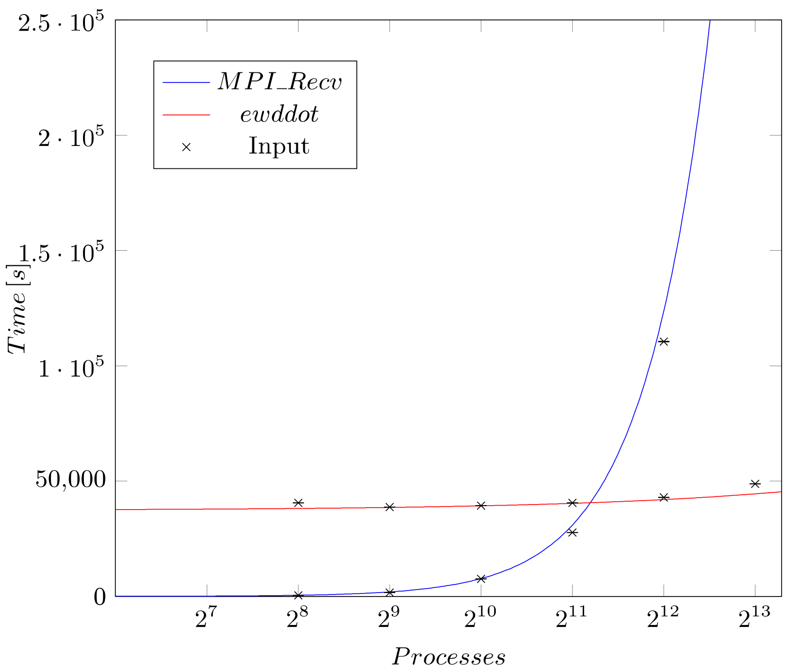
- **Finite element flow simulation program with numerous equations represented:**
  - Advection diffusion
  - Navier-Stokes
  - Shallow water



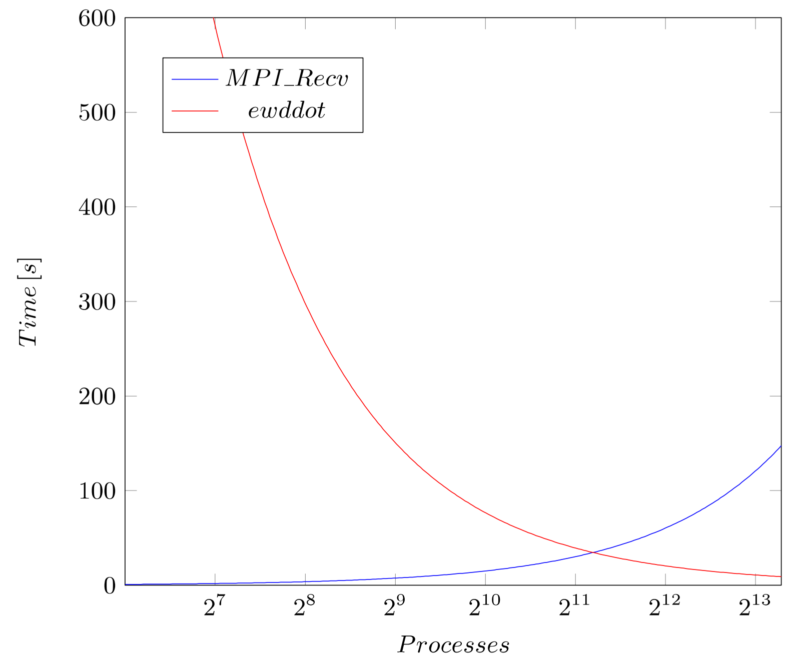
- **Strong scaling analysis**
  - P = {128; …; 4,096}
  - 5 measurements per $p_i$
  - Using accumulated time across processes as metric

# XNS (2)

Accumulated time




Wallclock time

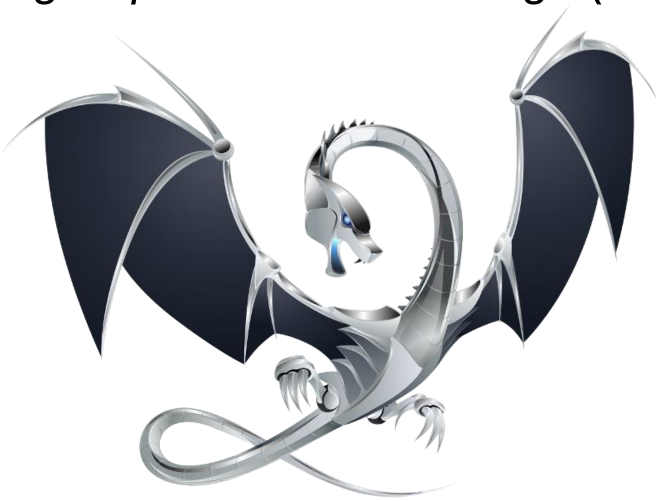



| Kernel | Runtime[%] p=128 | Runtime[%] p=4,096 | Model [s] t = f(p) |
|---|---|---|---|
| ewdgennprm->MPI_Recv | 0.46 | 51.46 | $0.029 \times p^2$ |
| ewddot | 44.78 | 5.04 | $p \times \log(p)$ |

#bytes = ~p
#msg = ~p

# Is this all? No, it’s just the beginning …

- **We face several problems:**
  - Multiparameter modeling – search space explosion
    *Interesting instance of the curse of dimensionality*
  - Modeling overheads
    *Cross validation (leave-one-out) is slow and*
    *Our current profiling requires a lot of storage (>TBs)*

# Step back – what do we really care about?

- **Work**

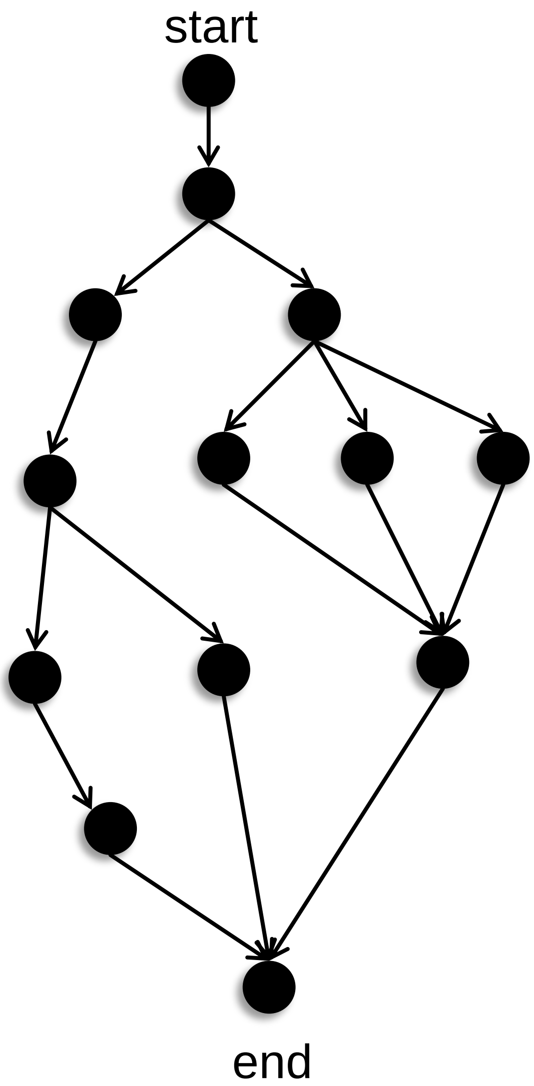
$$W = T_1$$

- **Depth**

$$D = T_\infty$$

- **Parallel efficiency**

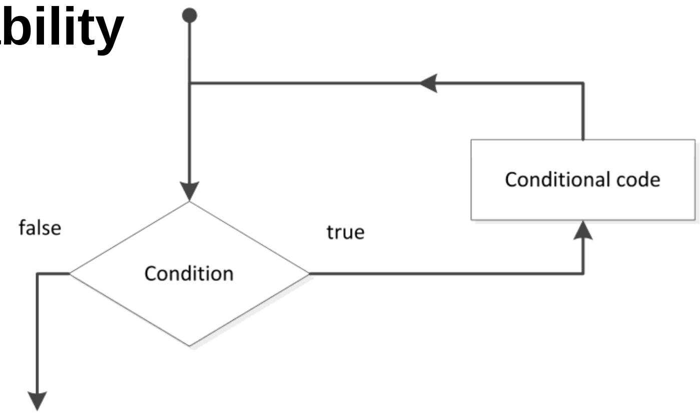
$$E_p = \frac{T_1}{pT_p}$$



# Static analysis of explicitly parallel programs

- **Structures that determine program scalability**

**LOOPS**



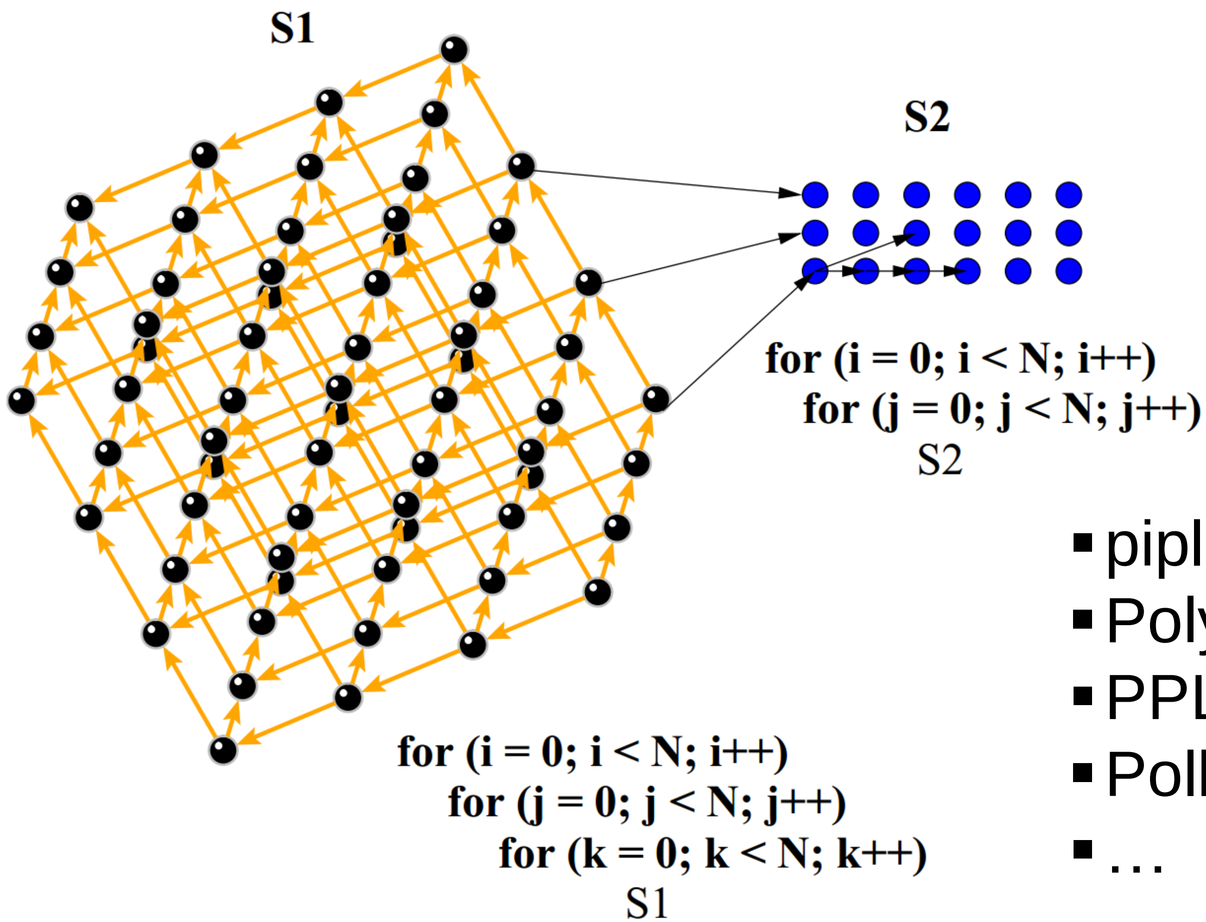

- **Assumption:**
  **Other instructions do not influence it**

- **Example:**

```
for (x=0; x < n/p; x++)
    for (y=1; y < n; y=2*y )
          veryComplicatedOperation(x,y);
```

# Related work: counting loop iterations

- **Polyhedral model**



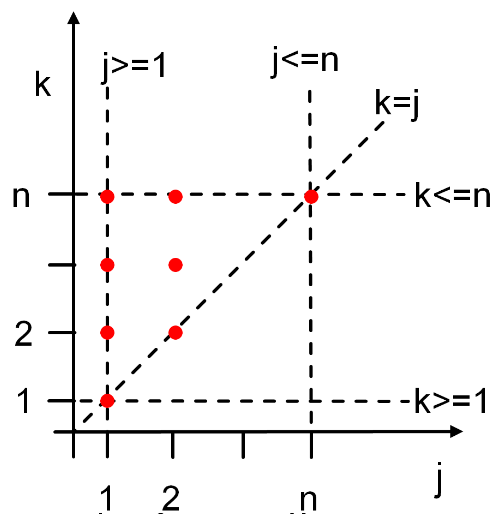

- piplib
- PolyLib
- PPL
- Polly
- …

R. Karp, R. Miller, and S. Winograd. The organization of computations for uniform recurrence equations. J. ACM, 14(3):563–590, July 1967.

# Related work: counting loop iterations

- **Polyhedral model**

```
for (j = 1; j <= n; j = j*2)
        for (k = j; k <= n; k = k++)
                veryComplicatedOperation(j,k);
```




$j \in$ [redacted] $, n$

$k \in$ [redacted] $, n$

$$N = (n+1)\log_2 n - n + 2$$

$$N = \frac{n(n+1)}{2}$$

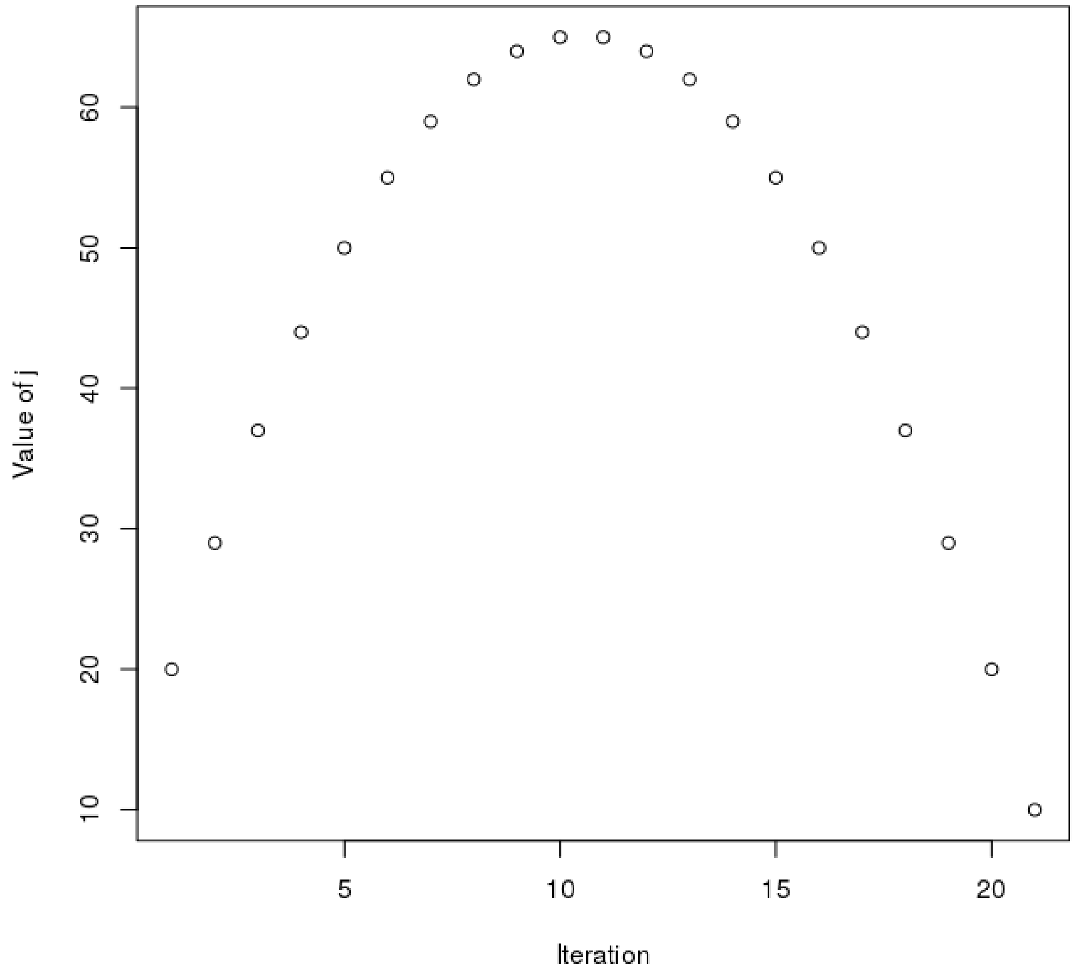
A.I. Barvinok. A polynomial time algorithm for counting integral points in polyhedra when the dimension is fixed, Math. Oper. Res., 1994

# Related work: counting loop iterations

- **When the polyhedral model cannot handle it**

```
j=10;
k=10;
while (j>0){
   j=j+k;
   k--;
}
```

?



# Counting arbitrary affine loop nests

- **Affine loops**

```
x=x_0;                 // Initial assignment
while(c^T x < g)       // Loop guard
    x=Ax + b;          // Loop update
```

- **Perfectly nested affine loops**

```
while(c_1^T x < g_1) {
  x = A_1 x + b_1;
  while(c_2^T x < g_2) {
    ...
     x = A_{k-1} x + b_{k-1};
     while(c_k^T x < g_k) {
        x = A_k x + b_k;
        while(c_{k+1}^T x < g_{k+1}) {... }
        x = U_k x + v_k; }
     x = U_{k-1} x + v_{k-1};
    ...}
  x = U_1 x + v_1;}
```

$$A_k, U_k \in \mathbb{R}^{m\times m},\ b_k, v_k, c_k \in \mathbb{R}^m,\ g_k \in \mathbb{R} \text{ and } k = 1 \ldots r.$$

# Counting arbitrary affine loop nests

- **Example**

```
for (j=1; j < n/p + 1; j= j*2)
        for (k=j; k < m; k = k + j )
                veryComplicatedOperation(j,k);
```

# Counting arbitrary affine loop nests

- **Example**

```
for (j=1; j < n/p + 1; j= j*2)
    for (k=j; k < m; k = k + j )
        veryComplicatedOperation(j,k);
```

**while**$(c_1^T x < g_1)$ {
  $x = A_1 x + b_1;$
  **while**$(c_2^T x < g_2)$ {
    ...
    $x = A_{k-1} x + b_{k-1};$
    **while**$(c_k^T x < g_k)$ {
      $x = A_k x + b_k;$
      **while**$(c_{k+1}^T x < g_{k+1})$ {... }
      $x = U_k x + v_k;$ }
    $x = U_{k-1} x + v_{k-1};$
  ...}
  $x = U_1 x + v_1;$}

# Counting arbitrary affine loop nests

- **Example**

```
for (j=1; j < n/p + 1; j= j*2)
        for (k=j; k < m; k = k + j )
                veryComplicatedOperation(j,k);
```

$$\begin{pmatrix} j \\ k \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} j \\ k \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix};$$

$$
\begin{array}{l}
\texttt{while}(c_1^T x < g_1)\ \{ \\
\quad x = A_1 x + b_1; \\
\quad \texttt{while}(c_2^T x < g_2)\ \{ \\
\qquad \ldots \\
\qquad x = A_{k-1} x + b_{k-1}; \\
\qquad \texttt{while}(c_k^T x < g_k)\ \{ \\
\qquad\quad x = A_k x + b_k; \\
\qquad\quad \texttt{while}(c_{k+1}^T x < g_{k+1})\ \{\ldots\ \} \\
\qquad\quad x = U_k x + v_k;\ \} \\
\qquad x = U_{k-1} x + v_{k-1}; \\
\quad \ldots\} \\
\quad x = U_1 x + v_1;\}
\end{array}
$$

# Counting arbitrary affine loop nests

- **Example**

```
for (j=1; j < n/p + 1; j= j*2)
    for (k=j; k < m; k = k + j )
        veryComplicatedOperation(j,k);
```

$$\begin{pmatrix} j \\ k \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} j \\ k \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix};$$

$$while\left(\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} j \\ k \end{pmatrix} < n/p + 1\right)\{$$

$$\}$$

---

$\texttt{while}(c_1^T x < g_1)\ \{$
$\quad x = A_1 x + b_1;$
$\quad \texttt{while}(c_2^T x < g_2)\ \{$
$\quad\quad \ldots$
$\quad\quad x = A_{k-1} x + b_{k-1};$
$\quad\quad \texttt{while}(c_k^T x < g_k)\ \{$
$\quad\quad\quad x = A_k x + b_k;$
$\quad\quad\quad \texttt{while}(c_{k+1}^T x < g_{k+1})\ \{\ldots\ \}$
$\quad\quad\quad x = U_k x + v_k;\ \}$
$\quad\quad x = U_{k-1} x + v_{k-1};$
$\quad\quad \ldots\}$
$\quad x = U_1 x + v_1;\}$

---

# Counting arbitrary affine loop nests

- **Example**

```
for (j=1; j < n/p + 1; j= j*2)
    for (k=j; k < m; k = k + j )
        veryComplicatedOperation(j,k);
```

$$\begin{pmatrix} j \\ k \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} j \\ k \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix};$$

$$while\left(\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} j \\ k \end{pmatrix} < n/p + 1\right)\{$$

$$\begin{pmatrix} j \\ k \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} j \\ k \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$while\left(\begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} j \\ k \end{pmatrix} < m\right)\{$$

$$\}$$

$$\}$$

---

while($c_1^T x < g_1$) {
  $x = A_1 x + b_1;$
  while($c_2^T x < g_2$) {
    ...
    $x = A_{k-1} x + b_{k-1};$
    while($c_k^T x < g_k$) {
      $x = A_k x + b_k;$
      while($c_{k+1}^T x < g_{k+1}$) {... }
      $x = U_k x + v_k;$ }
    $x = U_{k-1} x + v_{k-1};$
  ...}
  $x = U_1 x + v_1;$}

---

# Counting arbitrary affine loop nests

- **Example**

```
for (j=1; j < n/p + 1; j= j*2)
        for (k=j; k < m; k = k + j )
                veryComplicatedOperation(j,k);
```

$$\begin{pmatrix} j \\ k \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} j \\ k \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix};$$

$$while\left(\begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} j \\ k \end{pmatrix} < {}^{n}\!/\!_{p} + 1\right)\{$$

$$\begin{pmatrix} j \\ k \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} j \\ k \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$while\left(\begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} j \\ k \end{pmatrix} < m\right)\{$$

$$\begin{pmatrix} j \\ k \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} j \\ k \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$\}\begin{pmatrix} j \\ k \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} j \\ k \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$\}$$

---

$\texttt{while}(c_1^T x < g_1)\ \{$
  $x = A_1 x + b_1;$
  $\texttt{while}(c_2^T x < g_2)\ \{$
    $\ldots$
    $x = A_{k-1} x + b_{k-1};$
    $\texttt{while}(c_k^T x < g_k)\ \{$
      $x = A_k x + b_k;$
      $\texttt{while}(c_{k+1}^T x < g_{k+1})\ \{\ldots\ \}$
      $x = U_k x + v_k;\ \}$
    $x = U_{k-1} x + v_{k-1};$
  $\ldots\}$
  $x = U_1 x + v_1;\}$

---

# Counting arbitrary affine loop nests

- **Example**

```
for (j=1; j < n/p + 1; j= j*2)
    for (k=j; k < m; k = k + j )
        veryComplicatedOperation(j,k);
```

$$\texttt{while}(c_1^T x < g_1)\ \{$$
$$x = A_1 x + b_1;$$
$$\texttt{while}(c_2^T x < g_2)\ \{$$
$$\ldots$$
$$x = A_{k-1} x + b_{k-1};$$
$$\texttt{while}(c_k^T x < g_k)\ \{$$
$$x = A_k x + b_k;$$
$$\texttt{while}(c_{k+1}^T x < g_{k+1})\ \{\ldots\ \}$$
$$x = U_k x + v_k;\ \}$$
$$x = U_{k-1} x + v_{k-1};$$
$$\ldots\}$$
$$x = U_1 x + v_1;\}$$

$$x = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix};$$
$$while(\begin{pmatrix} 1 & 0 \end{pmatrix} x < n/p + 1)\{$$
$$x = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$
$$while(\begin{pmatrix} 0 & 1 \end{pmatrix} x < m)\{$$
$$x = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$
$$\}x = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$
$$\}$$

where $x = \begin{pmatrix} j \\ k \end{pmatrix}$

T. Hoefler, G. Kwasniewski: Automatic Complexity Analysis of Explicitly Parallel Programs, SPAA'14

# Overview of the whole system

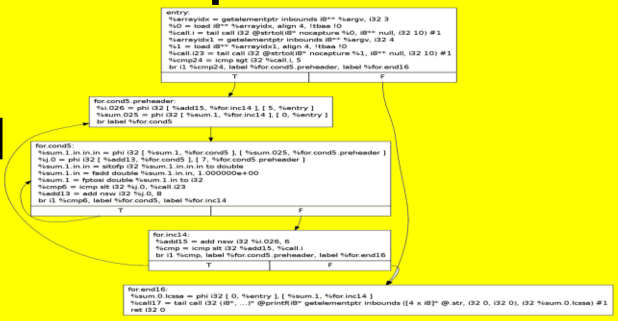
## Parallel program

```
do i = 1, procCols
   call mpi_irecv( buff, 2, dp_type, reduce_exch_proc(i),
>                  i, mpi_comm_world, request, ierr )
   call mpi_send( buff2, 2, dp_type, reduce_exch_proc(i),
>                  i, mpi_comm_world, ierr )
   call mpi_wait( request, status, ierr )
enddo

do i = id *n/p, ( id +1)* n/p
   do j = 1, nSize
       call compute
```

## LLVM

## Loop extraction




## Affine loop synthesis

$$
\begin{array}{l}
\texttt{while}(c_1^T x < g_1)\ \{ \\
\quad x = A_1 x + b_1; \\
\quad \texttt{while}(c_2^T x < g_2)\ \{ \\
\qquad \ldots \\
\qquad x = A_{k-1} x + b_{k-1}; \\
\qquad \texttt{while}(c_k^T x < g_k)\ \{ \\
\qquad\quad x = A_k x + b_k; \\
\qquad\quad \texttt{while}(c_{k+1}^T x < g_{k+1})\ \{\ldots\ \} \\
\qquad\quad x = U_k x + v_k;\ \} \\
\qquad x = U_{k-1} x + v_{k-1}; \\
\qquad \ldots \} \\
\quad x = U_1 x + v_1; \}
\end{array}
$$

## Closed form representation

$$x(i_1,\ldots,i_r) = A_{final}(i_1,\ldots,i_r)\cdot x_0 + b_{final}(i_1,\ldots,i_r)$$

with

$$i_r = 0\ldots n_k(x_{0,k}),\ k = 1\ldots r$$

## Number of iterations

$$N = \sum_{i_1=0}^{n_1(x_{0,1})} \sum_{i_2=0}^{n_2(x_{0,2})} \cdots \sum_{i_{r-1}=0}^{n_{r-1}(x_{0,r-1})} n_r(x_{0,r}).$$

## Program analysis

$$W = N\big|_{p=1}$$

$$D = N\big|_{p\to\infty}$$

# Algorithm in details

## Closed form representation of a loop

- Single affine statement

$$x = Lx + p$$

- Counting function

$$n(x_0)$$

$$x = x_0;$$
$$while\ (c^T x < g)$$
$$x = Ax + b;$$

$$x(i, x_0) = A^i x_0 + \sum_{j=0}^{i-1} A^j \cdot b$$

- Example

$$x(i, x_0) = L(i) \cdot x_0 + p(i)$$

$$n(x_0, c, g) = \arg\min_i (c^T \cdot x(i, x_0) \geq g)$$

$$x = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$
$$while\ ((1\ 0)\, x < m)\{$$
$$x = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$
$$\}$$

$$x(i, x_0) = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^i x_0 + \sum_{j=0}^{i-1} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^j \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix} x_0 + \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$n(x_0) = \left\lceil \frac{m - k_0}{j_0} \right\rceil$$

# Algorithm in details

## Folding the loops

$$x = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix};$$

$$\text{while } (\begin{pmatrix} 1 & 0 \end{pmatrix} x < n/p)\{$$

$$\quad x = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$\quad \text{while } (\begin{pmatrix} 0 & 1 \end{pmatrix} x < m)\{$$

$$\qquad x = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$\quad \} x = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$\}$$

# Algorithm in details

## Folding the loops

$$x = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix};$$

$$while\ (\begin{pmatrix} & 0 \end{pmatrix} x < n/p)\{$$

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$$\} x = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$\}$$

→

$$x = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix};$$

$$while\ (\begin{pmatrix} & 0 \end{pmatrix} x < n/p)\{$$

$$x = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$x = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$x = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$\}$$

# Algorithm in details

## Folding the loops

$$x = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix};$$

$while\ ((1 \;\; 0)x < n/p)\{$

$$x = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$while\ ((0 \;\; 1)x < m)\{$

$$x = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$\}\, x = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$\}$

$$x = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix};$$

$while\ ((1 \;\; 0)x < n/p)\{$

$$x = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$x = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$$x = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$\}$

$$x = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix};$$

$while\ ((1 \;\; 0)x < n/p)\{$

$$x = \begin{pmatrix} 2 & 0 \\ i+1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$\}$

# Algorithm in details

## Starting conditions

$x_{0,1} \longrightarrow$

$x = x_0;$

$while\ (c_1^T x < g_1)\{$

$x_{0,2} \longrightarrow$

$\quad x = A_1 x + b_1;$

$\quad while\ (c_2^T x < g_2)\{$

$x_{0,3} \longrightarrow$

$\quad\quad x = A_2 x + b_2;$

$\quad\quad while\ (c_3^T x < g_3)\{$

$\quad\quad\quad x = A_3 x + b_3;$

$\quad\quad \} x = U_2 x + v_2;$

$\quad \} x = U_1 x + v_1;$

$\}$

# Algorithm in details

## Counting the number of iterations

**We have:**

# Algorithm in details

## Counting the number of iterations

**We have:**

- The closed form for each loop:
  - *Single affine statement*
  - *Counting function*
- Starting condition for each loop

# Algorithm in details

## Counting the number of iterations

**We have:**

- The closed form for each loop:
  - *Single affine statement*
  - *Counting function*
- Starting condition for each loop

**Number of iterations:**

$$N = \sum_{i_1=0}^{n_1(x_{0,1})} \sum_{i_2=0}^{n_2(x_{0,2})} \dots \sum_{i_{r-1}=0}^{n_{r-1}(x_{0,r-1})} n_r(x_{0,r}).$$

# Algorithm in details

## Counting the number of iterations

- The equation computes the precise number of iterations

$$N = \sum_{i_1=0}^{n_1(x_{0,1})} \sum_{i_2=0}^{n_2(x_{0,2})} \dots \sum_{i_{r-1}=0}^{n_{r-1}(x_{0,r-1})} n_r(x_{0,r}).$$

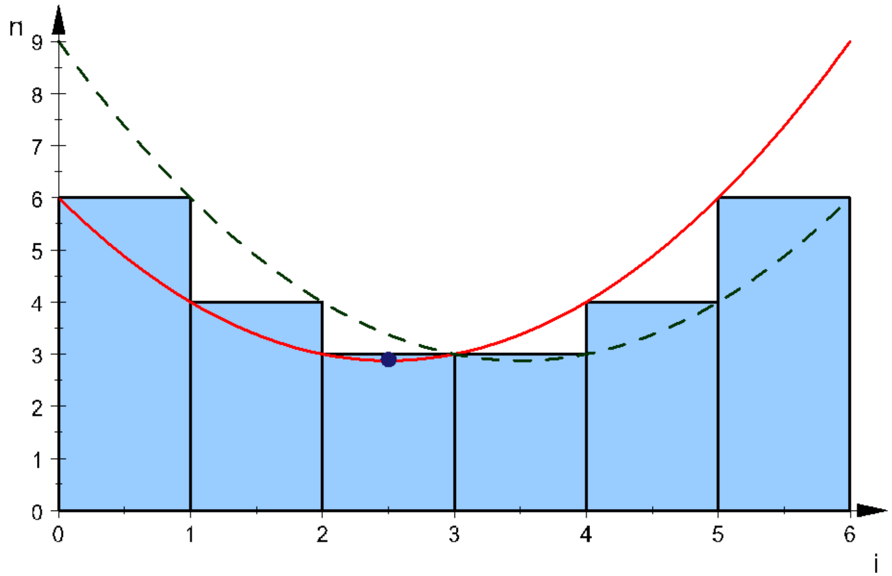
# Algorithm in details

## Counting the number of iterations

- The equation gives precise number of iterations

$$N = \sum_{i_1=0}^{n_1(x_{0,1})} \sum_{i_2=0}^{n_2(x_{0,2})} \dots \sum_{i_{r-1}=0}^{n_{r-1}(x_{0,r-1})} n_r(x_{0,r}).$$

- But simplification may fail → Sum approximation
  - *Approximate sums by integrals*
    → lower and upper bounds




T. Hoefler, G. Kwasniewski: Automatic Complexity Analysis of Explicitly Parallel Programs, SPAA'14

# Solving more general problems

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- **Multipath loops**

# Solving more general problems

- Multipath loops
- Conditional statements

# Solving more general problems

- Multipath loops
- Conditional statements
- Non-affine loops

```
do j=1,lastrow-firstrow+1
   sum = 0.d0
   do k=rowstr(j),rowstr(j+1)-1
      sum = sum + a(k)*p(colidx(k))
   enddo
   w(j) = sum
enddo
```

$\texttt{lastrow-firstrow+1} = \texttt{row\_size} = \frac{\texttt{na}}{\texttt{nprows}}$

$\texttt{rowstr(j+1)-1-rowstr(j)} = u$

$$N = \frac{\texttt{na} \cdot u}{\texttt{nprows}}$$

# Case studies

- **NAS Parallel Benchmarks: EP**

$$N(m,p) = \left\lceil \frac{2^{m-16} \cdot (u + 2^{16})}{p} \right\rceil$$

```
u :   do i=1,100
        ik =kk/2
        if (ik .eq. 0) goto 130
        kk=ik
      continue
```

# Case studies

- **NAS Parallel Benchmarks: EP**

$$N(m,p) = \left\lceil \frac{2^{m-16} \cdot (u + 2^{16})}{p} \right\rceil$$

$u$:

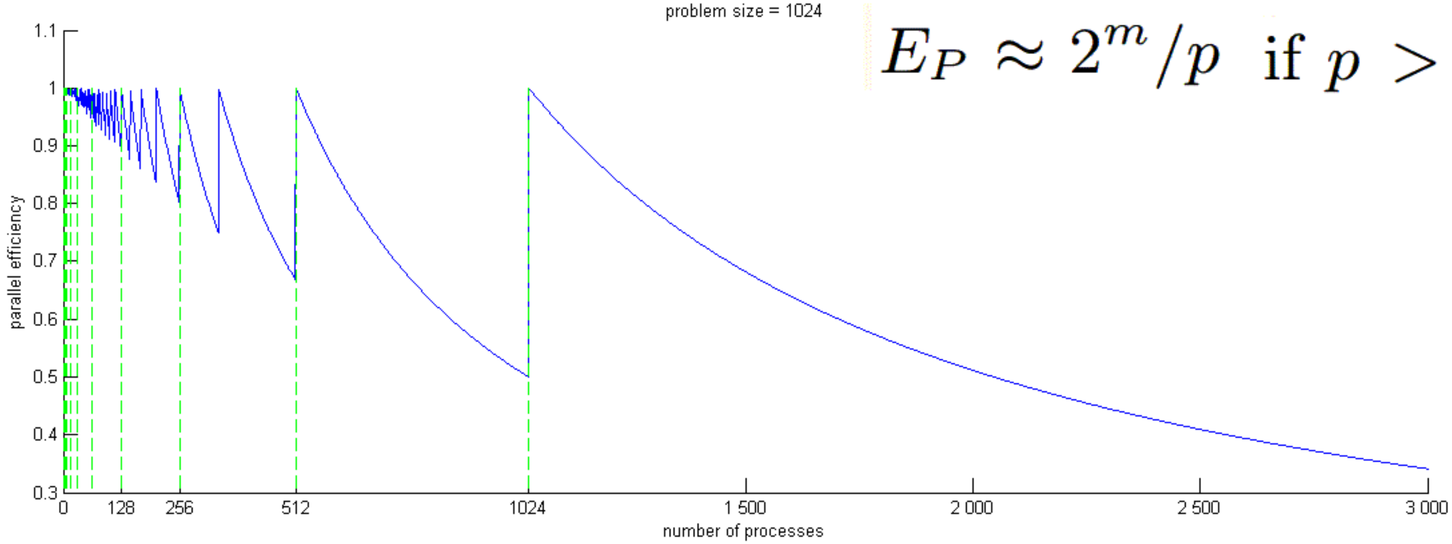
```
do i=1,100
  ik =kk/2
  if (ik .eq. 0) goto 130
  kk=ik
continue
```

$$W = T_1 \approx 2^m$$
$$D = T_\infty \approx 1$$

$$E_P = \frac{2^m}{p\left\lceil \frac{2^m}{p} \right\rceil}$$

$$E_P \approx 1 \text{ if } p \leq 2^m$$

$$E_P \approx 2^m/p \;\text{ if } p > 2^m$$



# Case studies

## CG – conjugate gradient

$$W \approx k_1 \left\lceil \frac{m}{p} \right\rceil + k_2 \sqrt{\left\lceil \frac{m}{p} \right\rceil} + k_3 \log_2\left(\sqrt{p}\right)$$

$$D = T_\infty = \infty$$

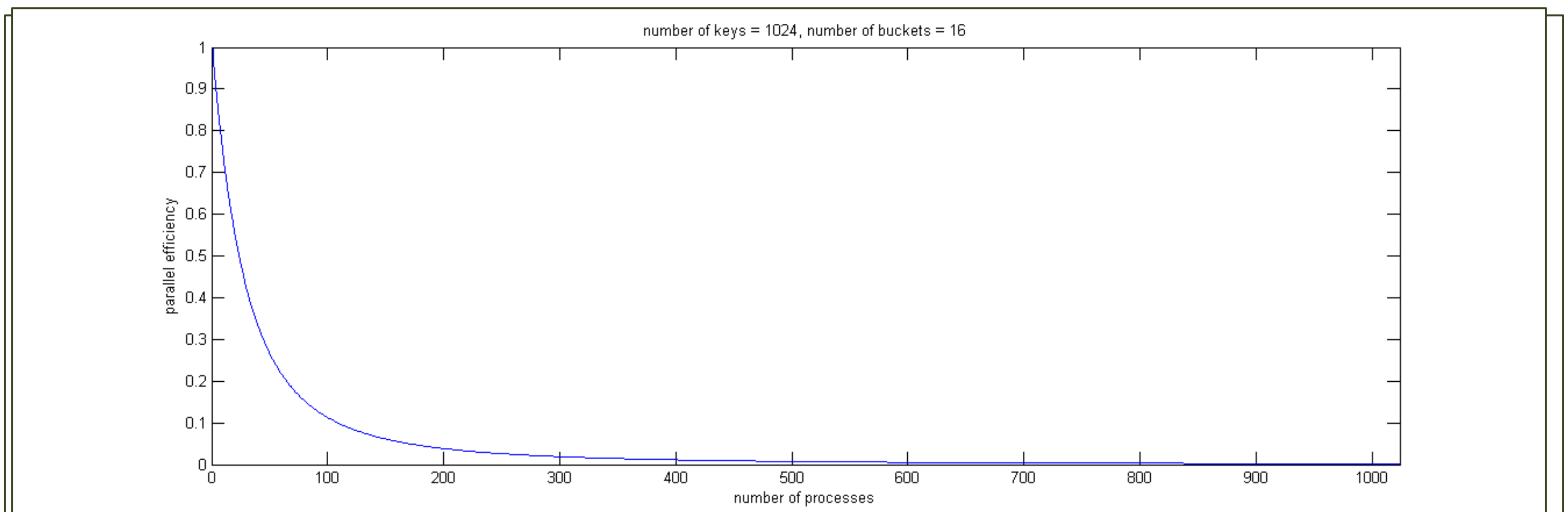
$$E_p = \frac{k_4}{p\left(k_1 \left\lceil \frac{m}{p} \right\rceil + k_2 \sqrt{\left\lceil \frac{m}{p} \right\rceil} + k_3 \log_2\left(\sqrt{p}\right)\right)}$$

## IS – integer sort

$$W \approx n\left(3(k + t) + 2\left\lceil \frac{m}{p} \right\rceil + p + u_1 + u_2\right)$$

$$D = T_\infty = \infty$$

$$E_p = \frac{k_1}{p\left(p + k_2 \sqrt{\left\lceil \frac{m}{p} \right\rceil} + p k_3\right)}$$



# What problems are remaining?

- **Well, what about non-affine loops?**
  - More general abstract interpretation (next step)
  - Not solvable → will always have undefined terms

$$N = \frac{\mathtt{na} \cdot u}{\mathtt{nprows}}$$

- **Back to PMNF?**
  - Generalize to multiple input parameters

    *a) Bigger search-space ☹*

    *b) Bigger trace files ☹*

- **Ad-hoc (partial) solution: online machine learning – PEMOGEN**
  - Replace cross-validation with LASSO (regression with $L_1$ regularizer)

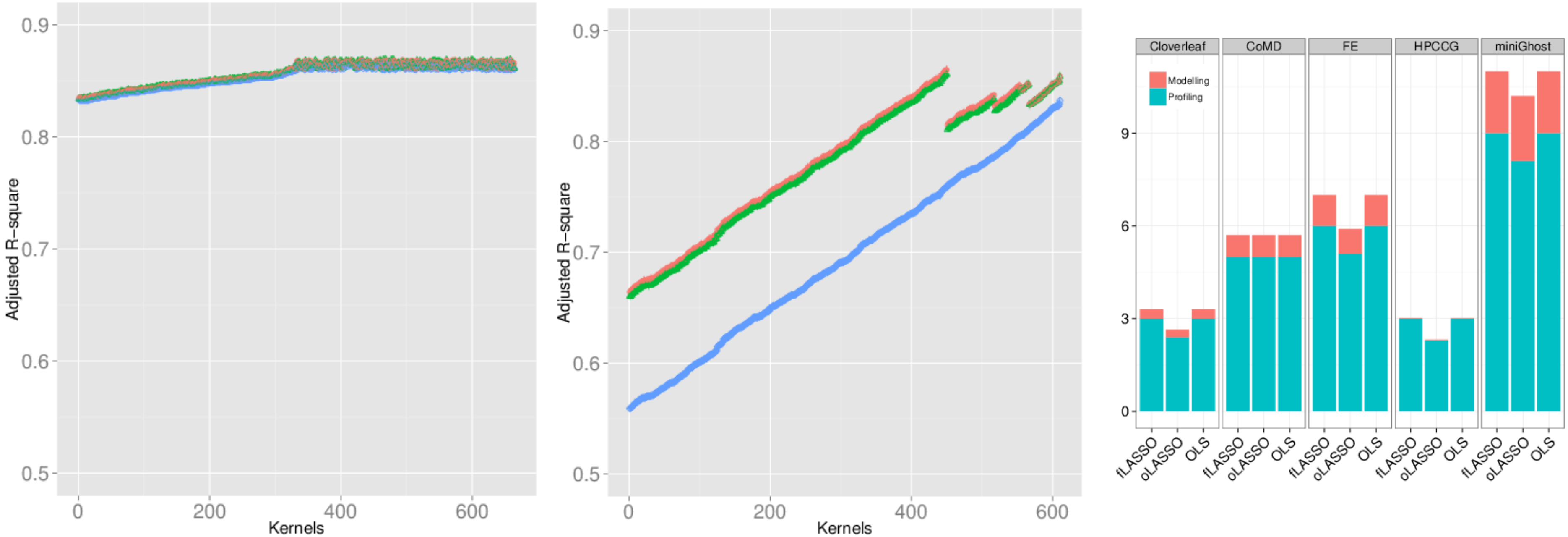
    *Much cheaper!*
  - Replace LASSO with online LASSO [1]

    *No traces!*

P. Garrigues and L. El Ghaoui. An homotopy algorithm for the Lasso with online observations. NIPS 2008

# PEMOGEN – static analysis

- **Also integrated into LLVM compiler**
  - Automatic kernel detection and instrumentation (Loop Call Graph)
  - Static dataflow analysis reduces parameter space for each kernel

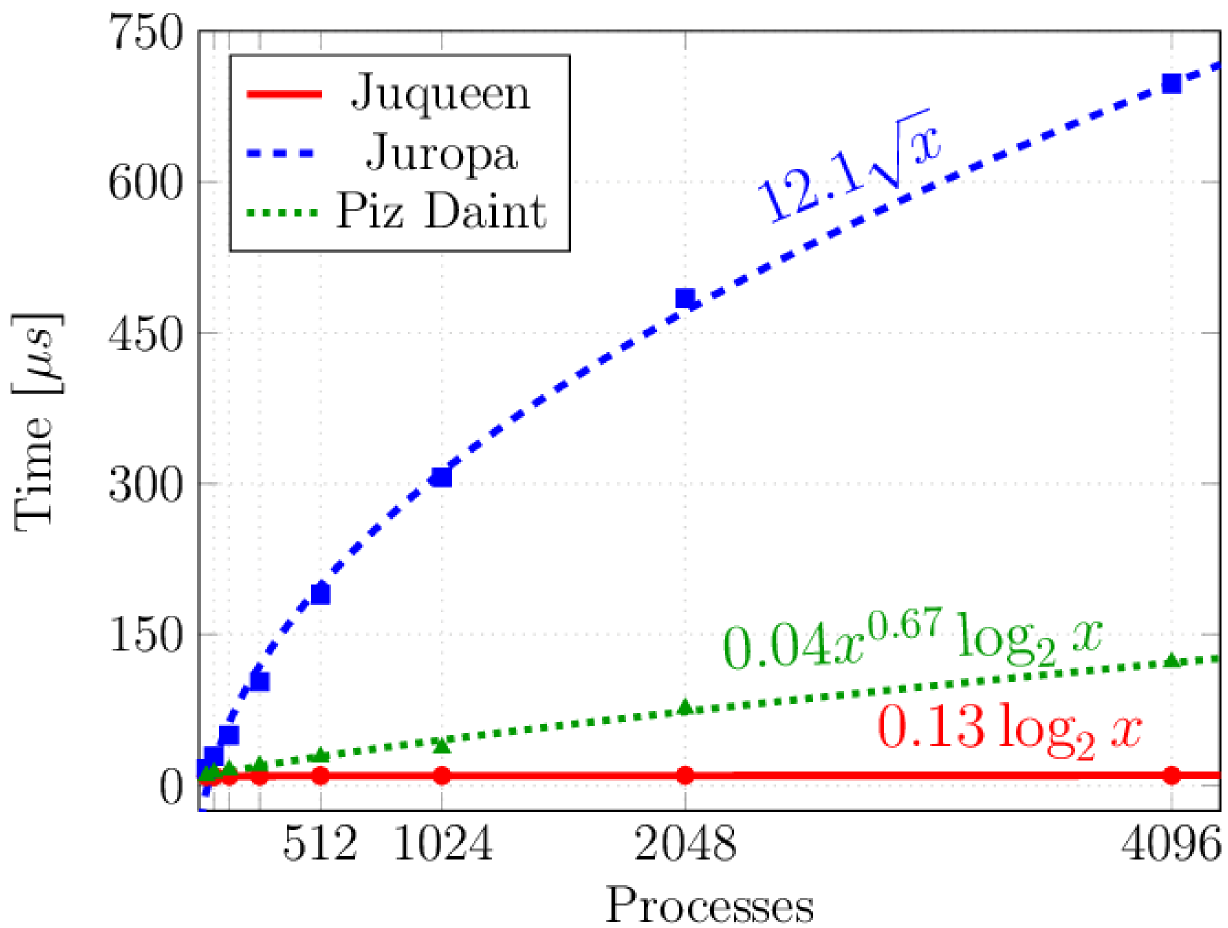



Quality: NAS UA and Mantevo MiniFE

Overhead: Mantevo

A. Bhattacharyya, T. Hoefler: PEMOGEN: Automatic Adaptive Performance Modeling during Program Runtime, PACT'14

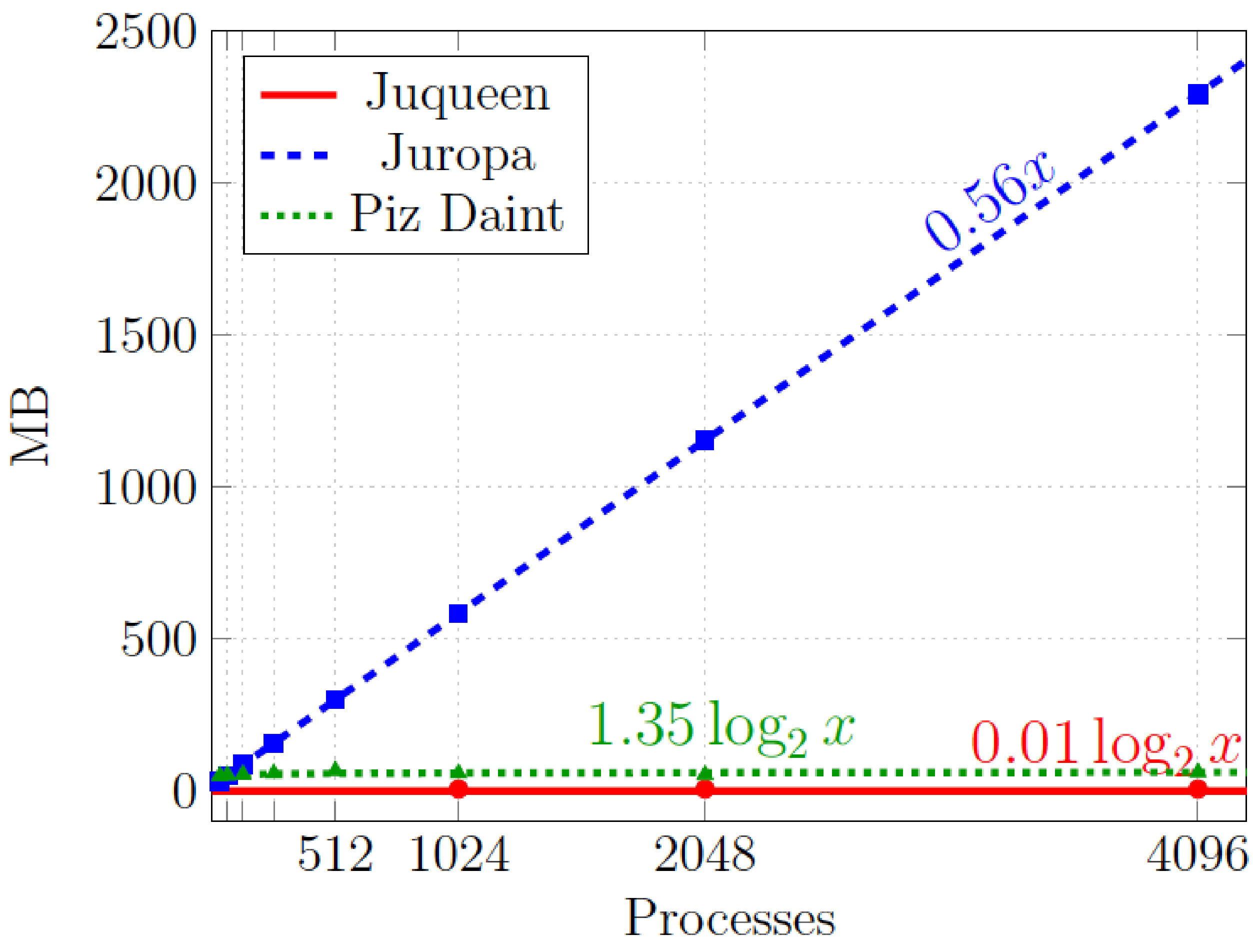
# Use-case A: automatic testing (Allreduce time)




- **Divergence on Piz**
- **Daint is $O(p^{0.67})$, the highest of all three**

S. Shudler, A. Calotoiu, T. Hoefler, A. Strube, F. Wolf: Exascaling Your Library: Will Your Implementation Meet Your Expectations?, ICS'15

# Use-case B: automatic testing (MPI memory size)




- **Linear memory consumption on Juropa**
- **ParaStation MPI**
- **uses RC over IB**

S. Shudler, A. Calotoiu, T. Hoefler, A. Strube, F. Wolf: Exascaling Your Library: Will Your Implementation Meet Your Expectations?, ICS'15

# Performance Analysis 2.0 – Automatic Models

- **Is feasible**
  ***Still a long way to go …***
- **Offers insight**
- **Requires low effort**
- **Improves code coverage**

SPPEXA

DFG

FNSNF

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

German Research School
for Simulation Sciences

SPAA 2014

SC13 Denver, CO 2013

ICS

A. Calotoiu, T. Hoefler, M. Poke, F. Wolf: Using Automated Performance Modeling to Find Scalability Bugs in Complex Codes. *Supercomputing (SC13).*

T. Hoefler, G. Kwasniewski: Automatic Complexity Analysis of Explicitly Parallel Programs. *SPAA 2014.*

A. Bhattacharyya, T. Hoefler: PEMOGEN: Automatic Adaptive Performance Modeling during Program Runtime, *PACT 2014*

S. Shudler, A. Calotoiu, T. Hoefler, A. Strube, F. Wolf: Exascaling Your Library: Will Your Implementation Meet Your Expectations? *ICS 2015*

# Backup

# Counting Arbitrary Affline Loop Nests

- **Why affine loops?**
  - Closed form representation of the loop

```
x=x0;              // Initial assignment
while(c^T x < g)   // Loop guard
    x=Ax + b;      // Loop update
```

$$x(i, x_0) = L(i) \cdot x_0 + p(i)$$

$$n(x_0, c, g) = \arg\min_{d} (c^T \cdot x(d, x_0) \geq g)$$

# Counting Arbitrary Affine Loop Nests

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- **Example**

```
for ( k=j; k < m; k = k + j )
        veryComplicatedOperation(j,k);
```

$$x = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$while \; ( (0 \;\; 1) x < m ) \{$

$$x = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix};$$

$\}$

$$x(i, x_0) = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix} x_0 + \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$n(x_0) = \left\lceil \frac{m - k_0}{j_0} \right\rceil$$

where $x_0 = \begin{pmatrix} j_0 \\ k_0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

# Loops

- **Multipath affine loops**

```
x=1;
while(x < n/p + 1) {
  y=x;
  while(y < m) { S1; y=2*y; }
  z=x;
  while(z < m) { S2; z= z + x; }
  x=2*x;
}
```