

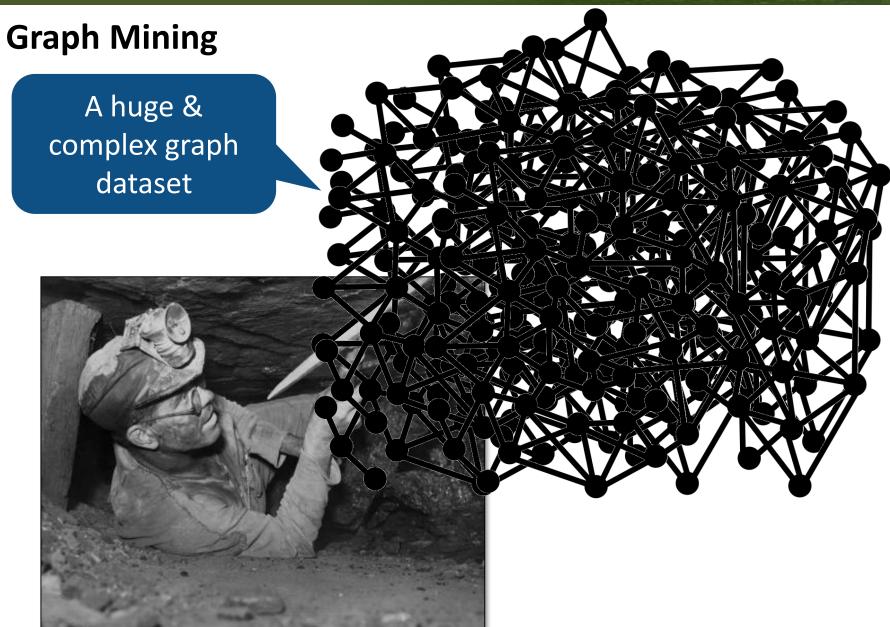
## **ProbGraph: High-Performance and High-Accuracy Graph Mining with Probabilistic Set Representations**

M. BESTA, C. MIGLIOLI, P. S. LABINI, J. TĚTEK, P. IFF, R. KANAKAGIRI, S. ASHKBOOS, K. JANDA, M. PODSTAWSKI, G. KWASNIEWSKI, N. GLEINIG, F. VELLA, O. MUTLU, T. HOEFLER.



## **Graph Mining**







**Graph Mining** 

A huge &

complex graph

dataset



Pattern counting (triangles, higherorder cliques, dense subgraphs, ...)



**Graph Mining** 

A huge &

complex graph

dataset

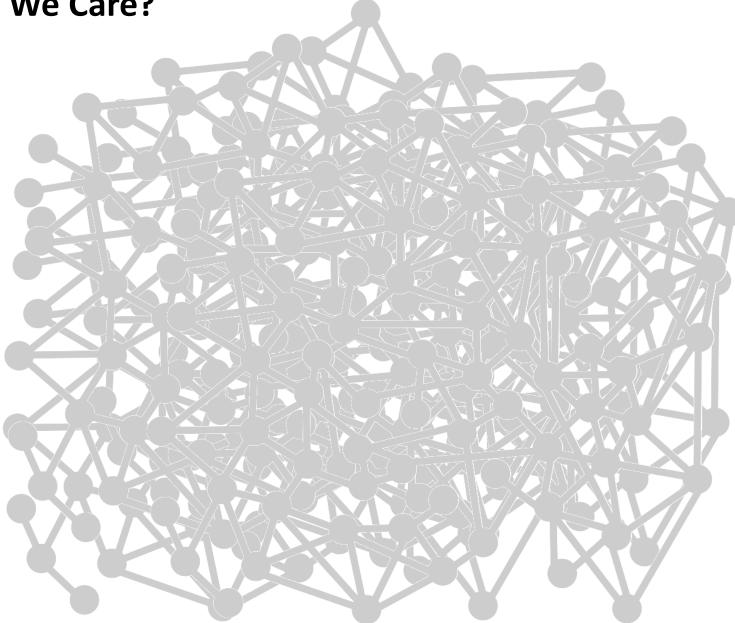


Pattern counting (triangles, higherorder cliques, dense subgraphs, ...)

Clustering, Link Prediction, Vertex Similarity, ...

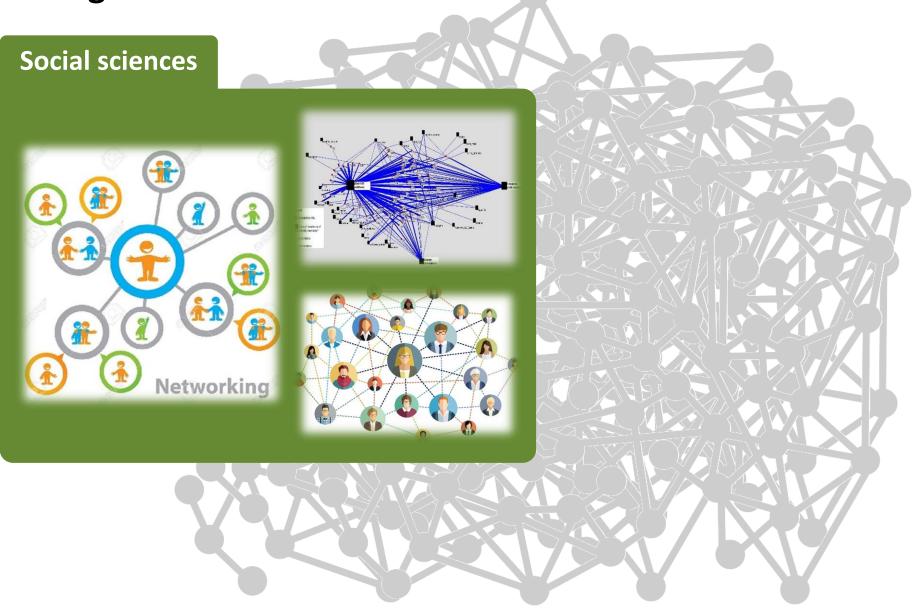






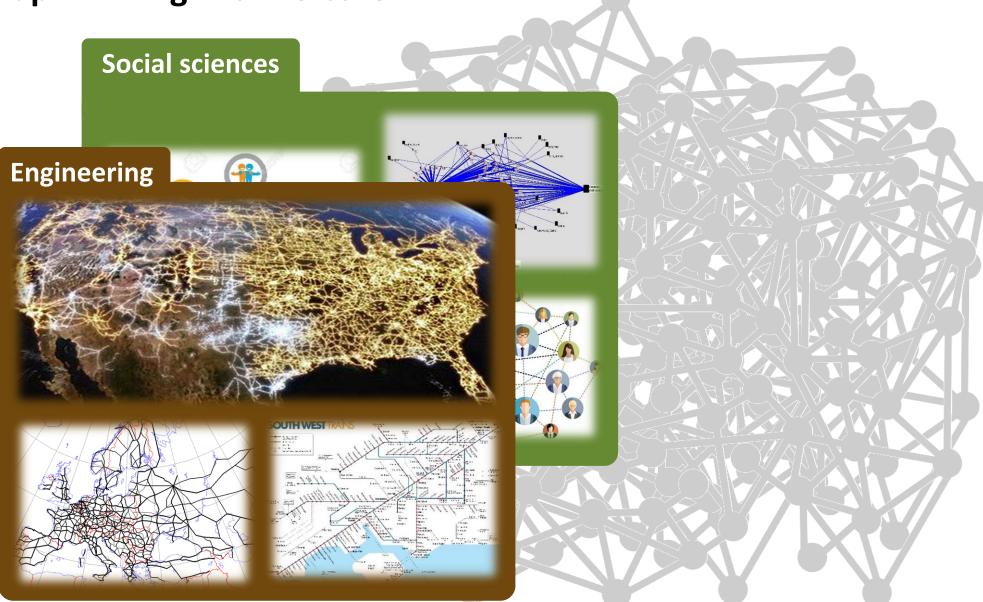
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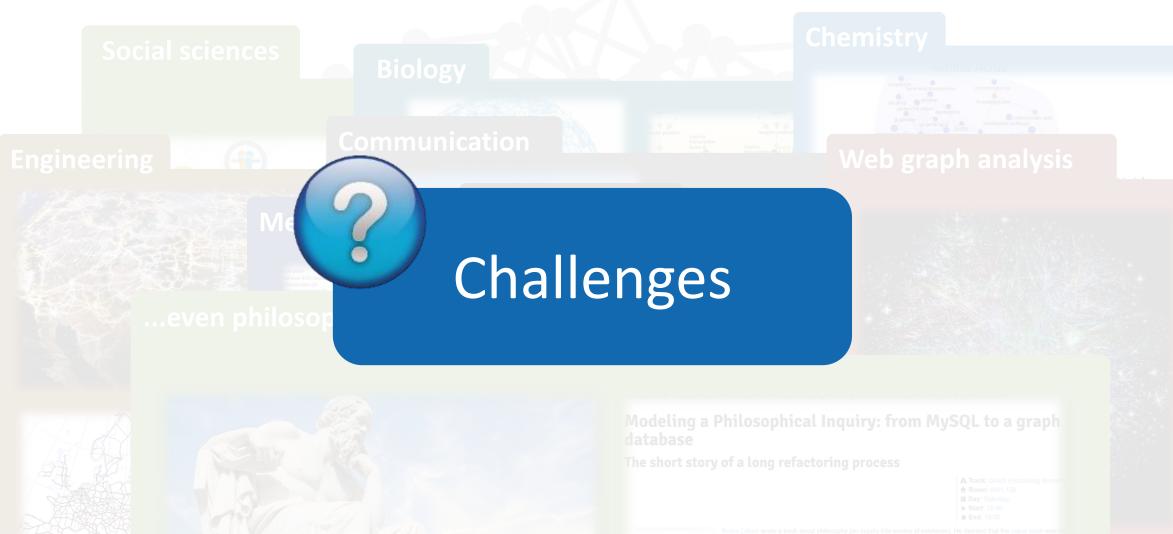




## **Graph Mining: Do We Care?**





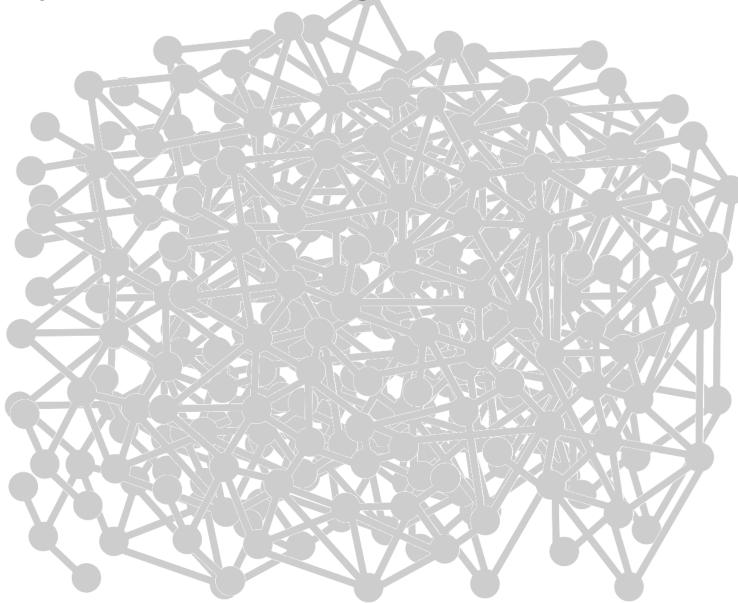


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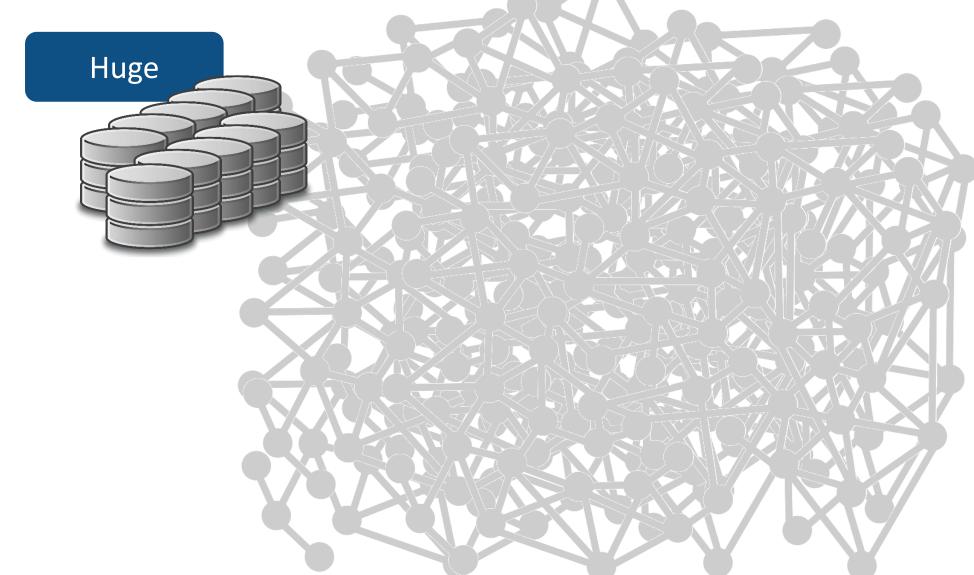


## **Graph Mining & Graph Datasets: Challenges**



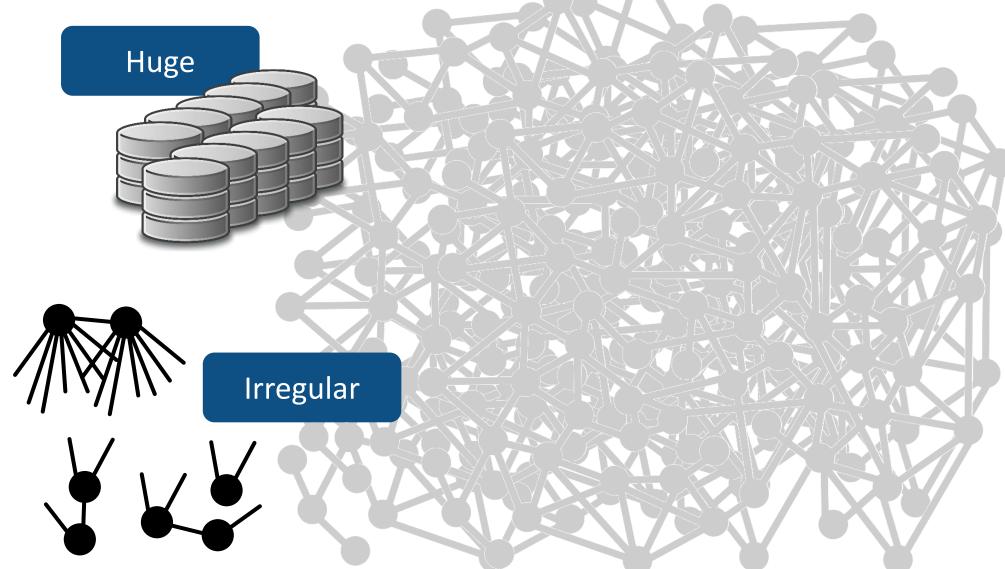


## **Graph Mining & Graph Datasets: Challenges**

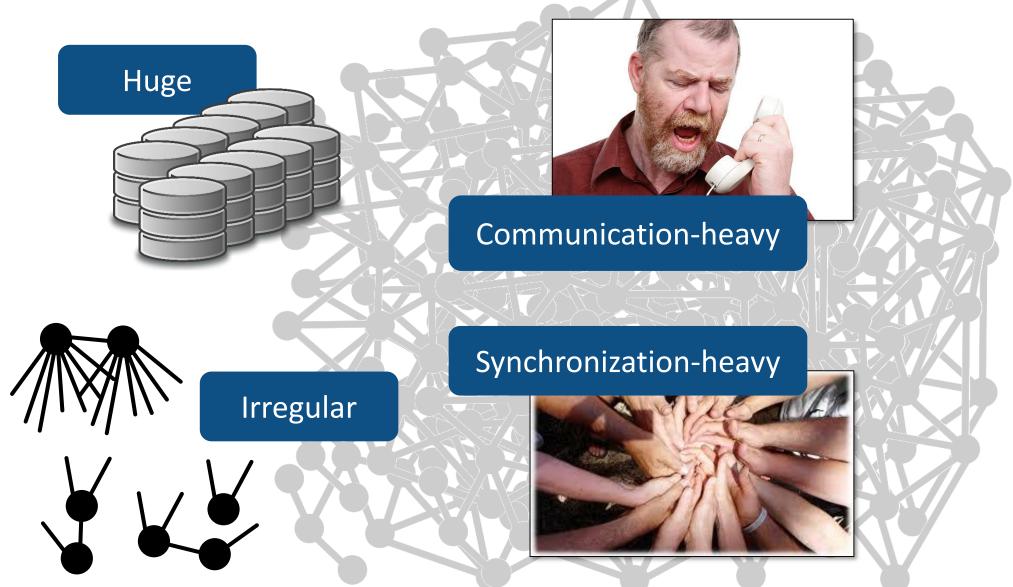




### **Graph Mining & Graph Datasets: Challenges**



## **Graph Mining & Graph Datasets: Challenges**



Huge

## **Graph Mining & Graph Datasets: Challenges**

Irregular



### Communication-heavy

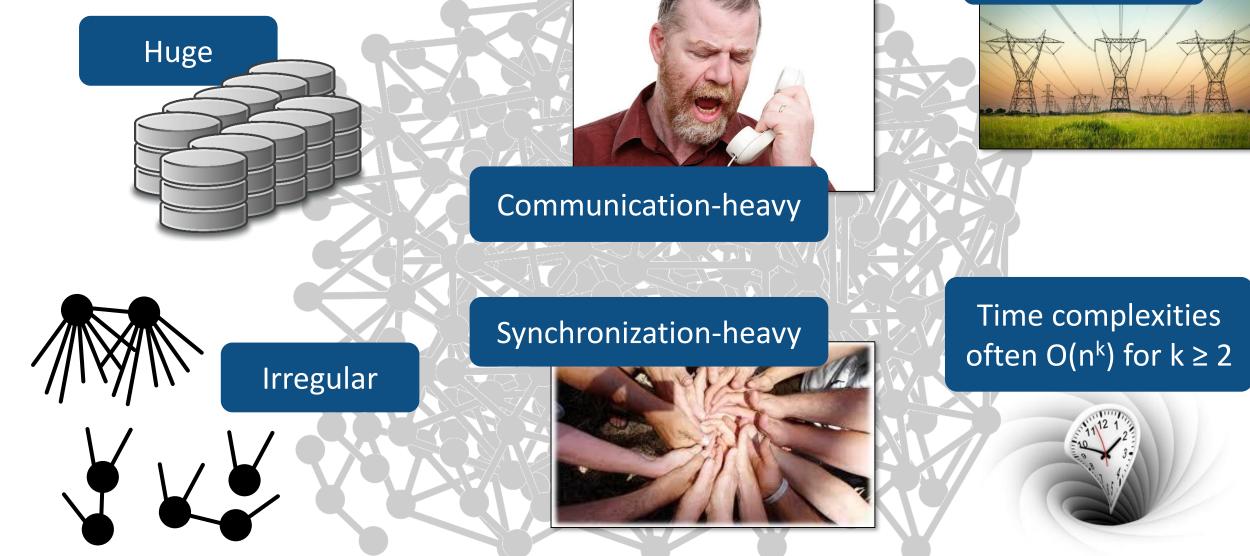
## Power-hungry



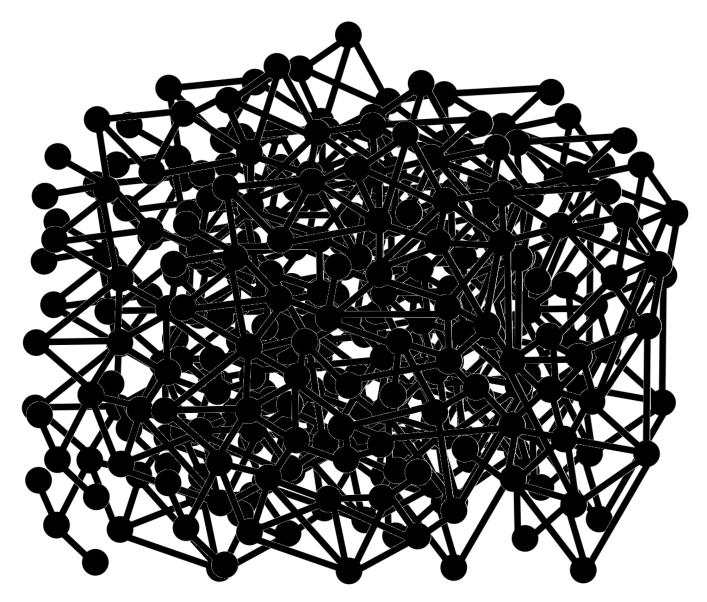
### Synchronization-heavy

## **Graph Mining & Graph Datasets: Challenges**

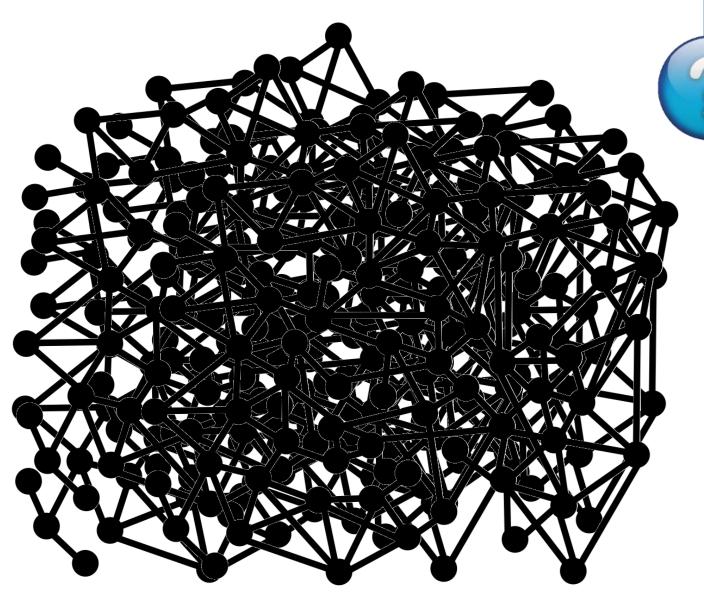






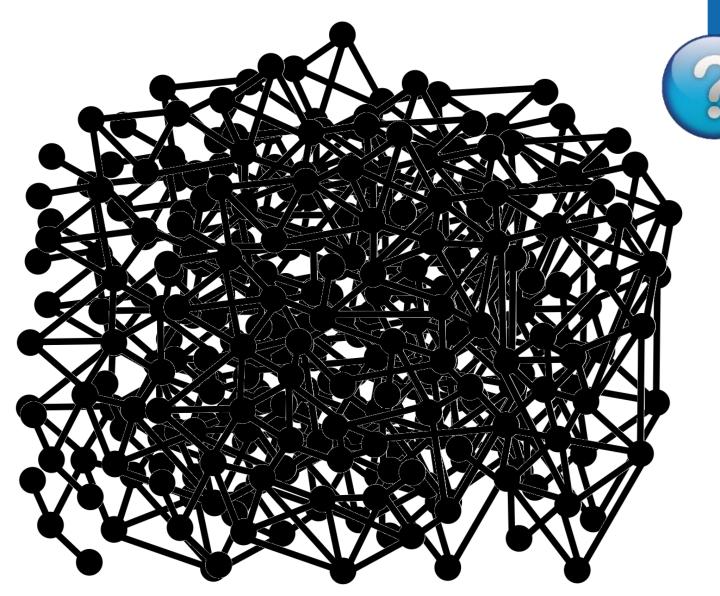






## Do we need 100% accurate results in all cases?





## Do we need 100% accurate results in all cases?

Let's say we can choose between...

Find all the patterns (e.g., cliques) in 1 day

Find ≥ 90% of all the patterns in 30 minutes





The choice is yours

**Choose wisel** 

# Do we need 100% accurate results in all cases?

Let's say we can choose between...

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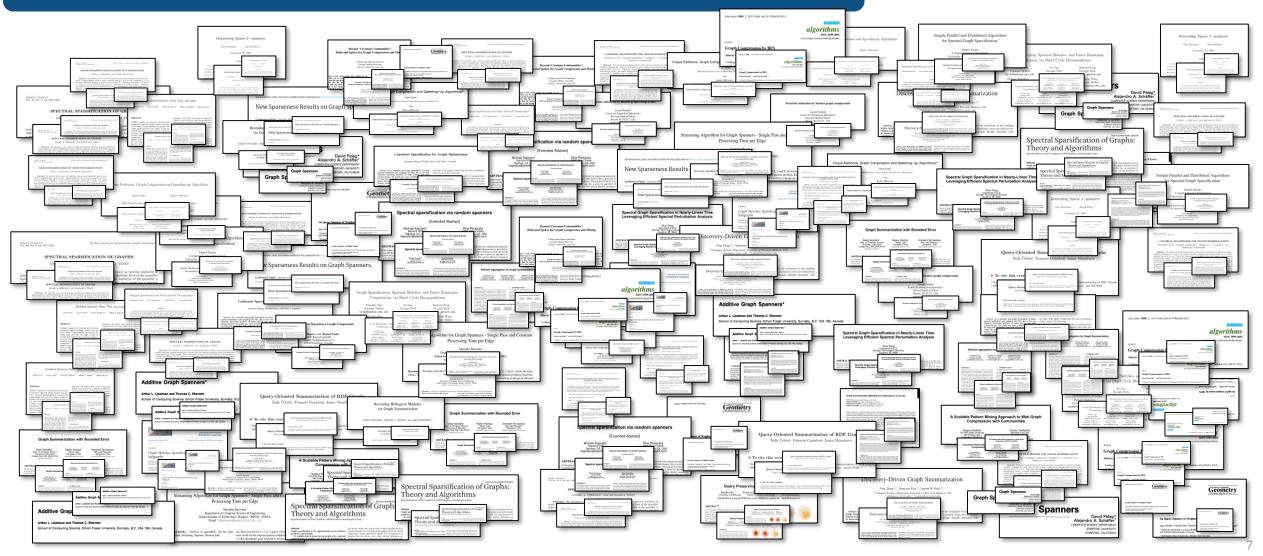


Start Start Barris Start Start

### **SPCL**

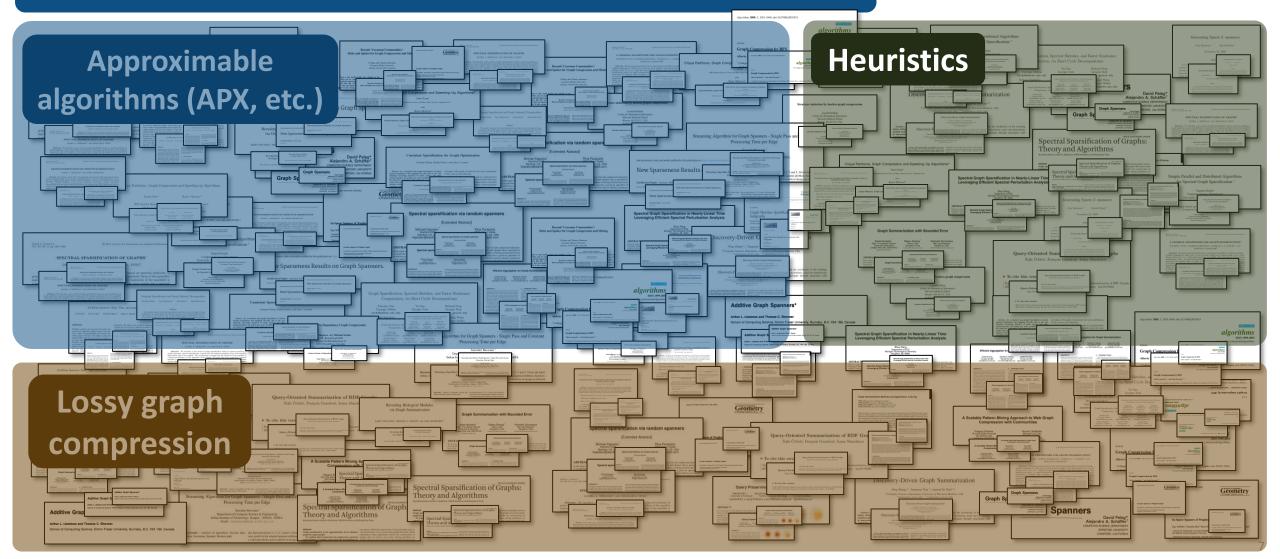
## **Approximate Graph Processing: State & Challenges**

### We analyzed > 500 works and identified three classes of schemes...



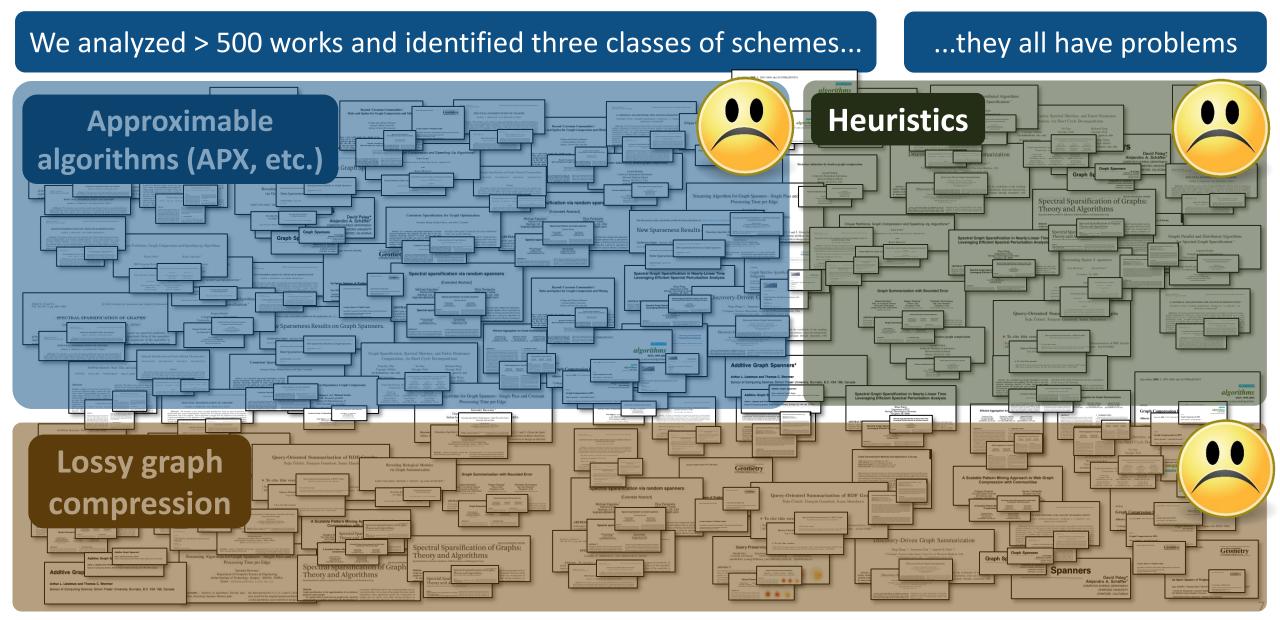


### We analyzed > 500 works and identified three classes of schemes...



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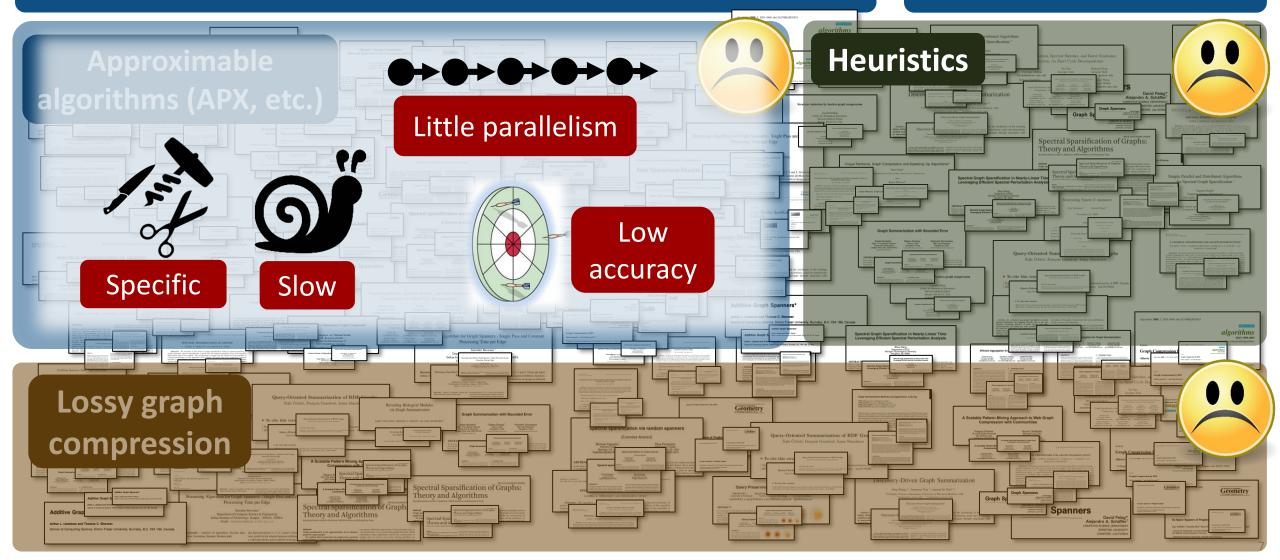


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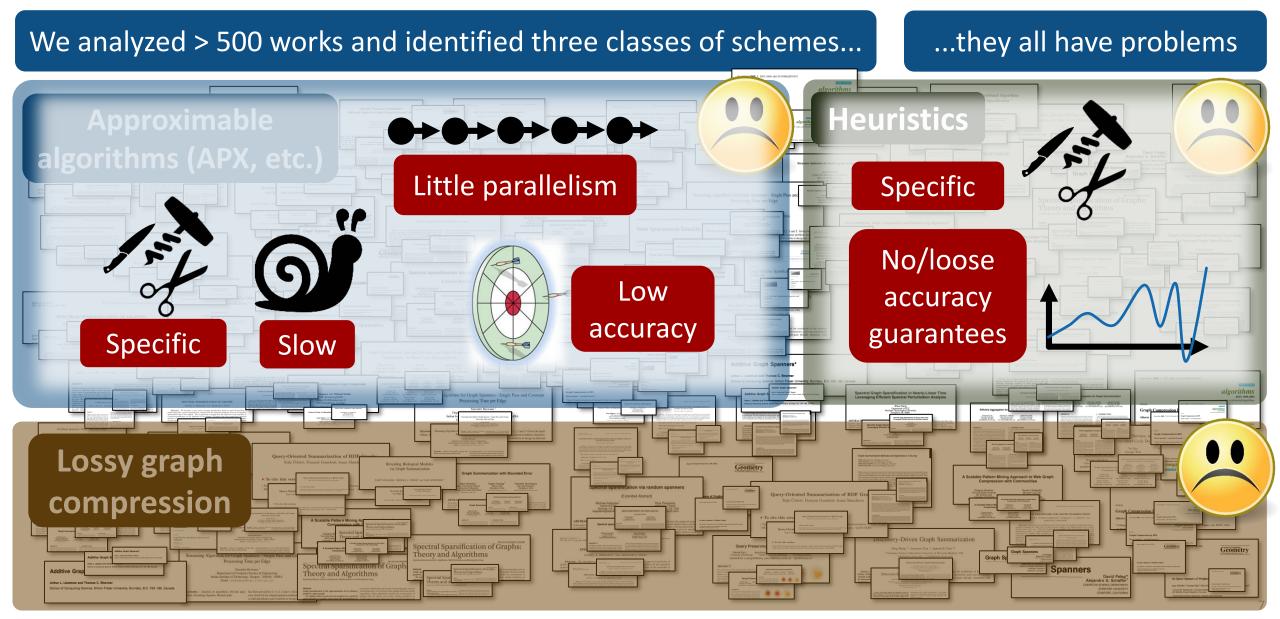
### We analyzed > 500 works and identified three classes of schemes...

#### ...they all have problems



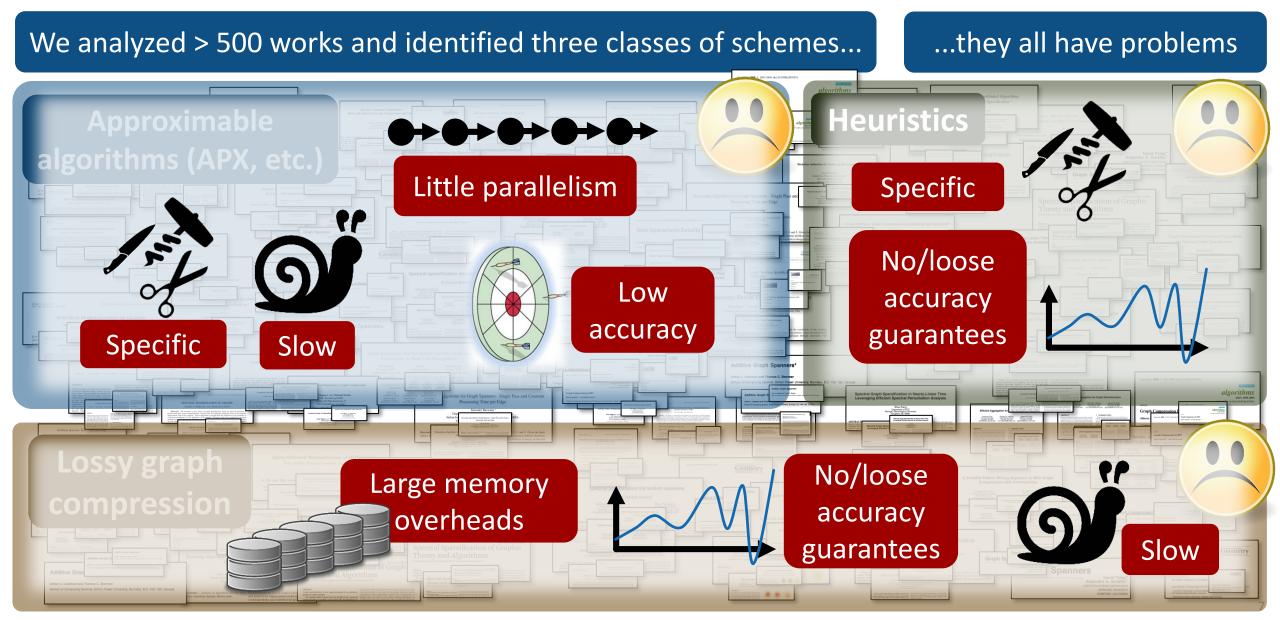
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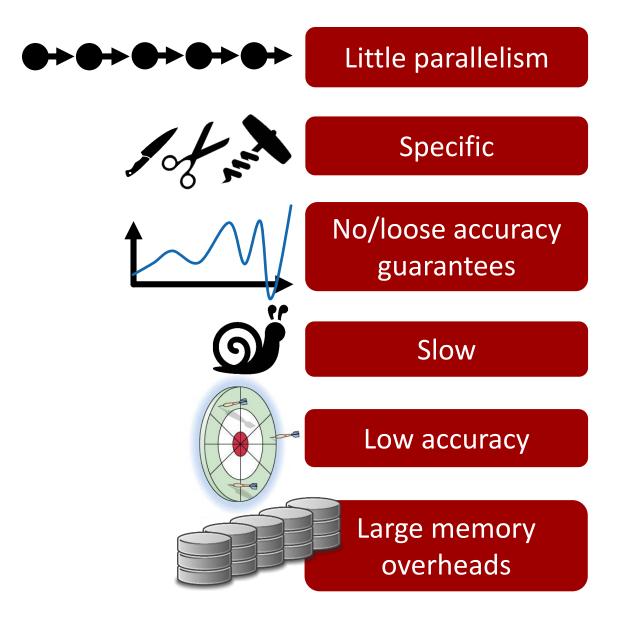
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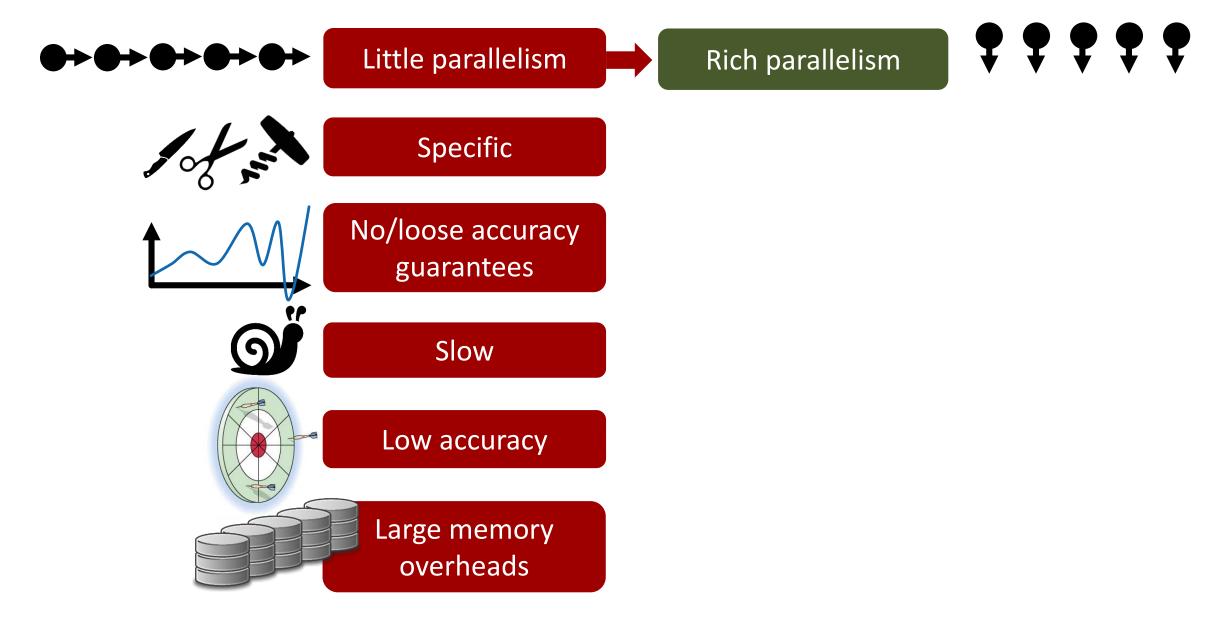
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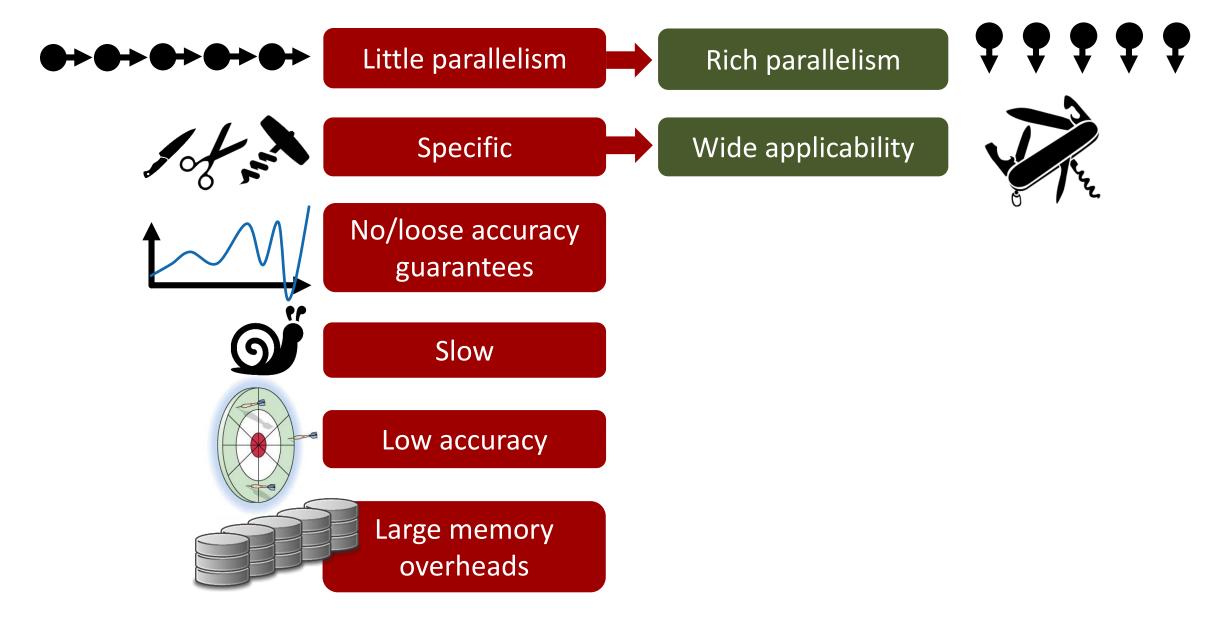






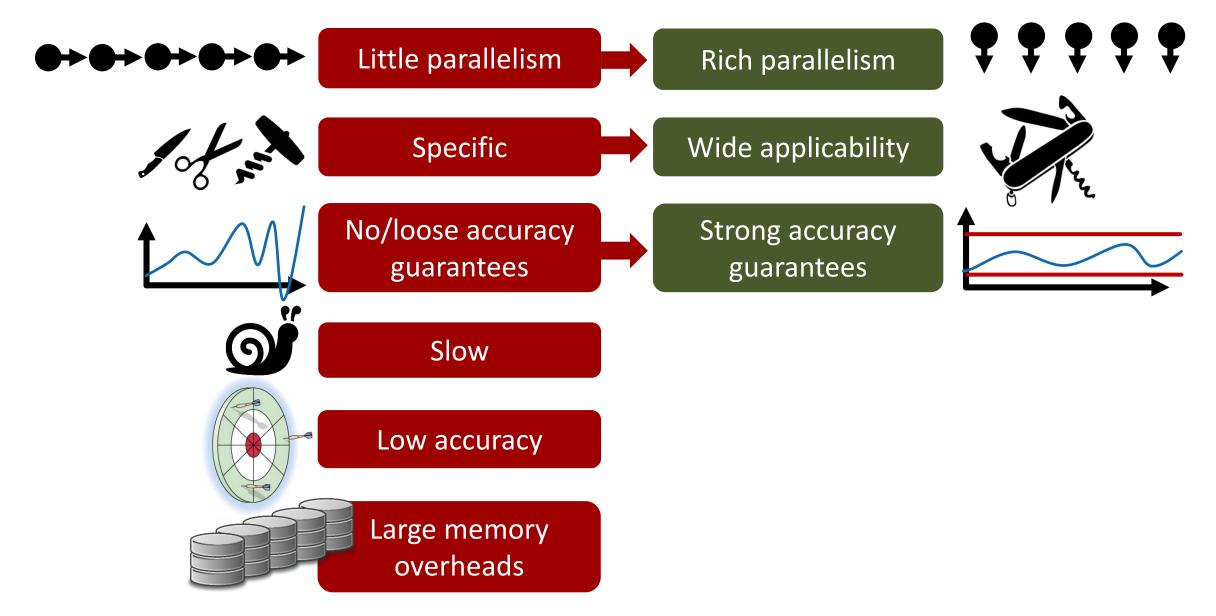
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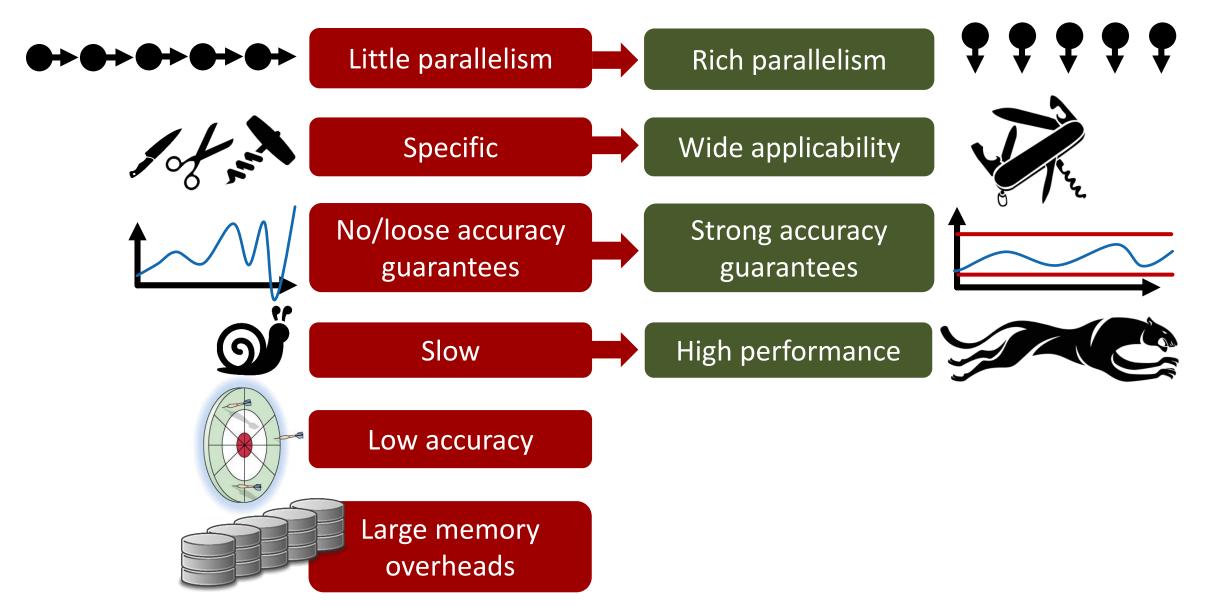
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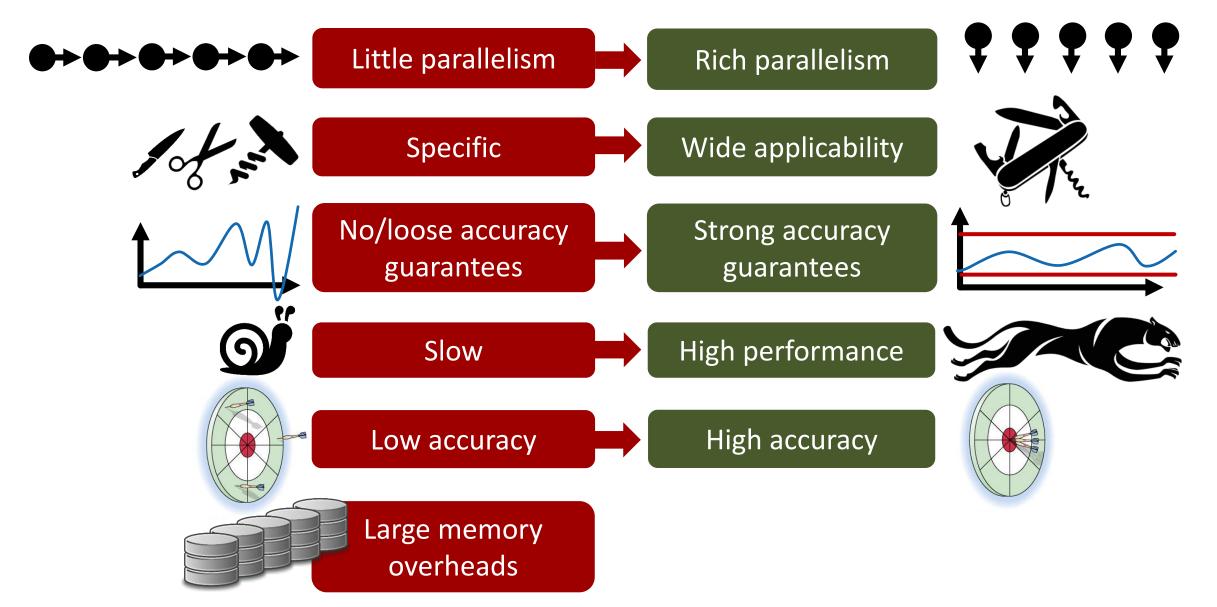
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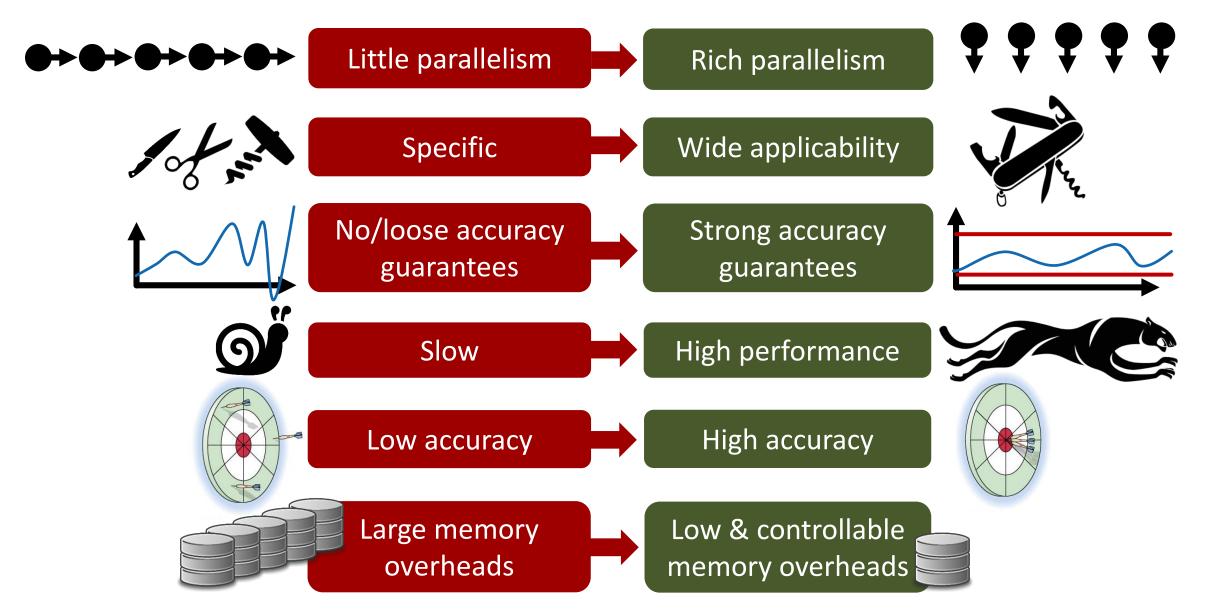
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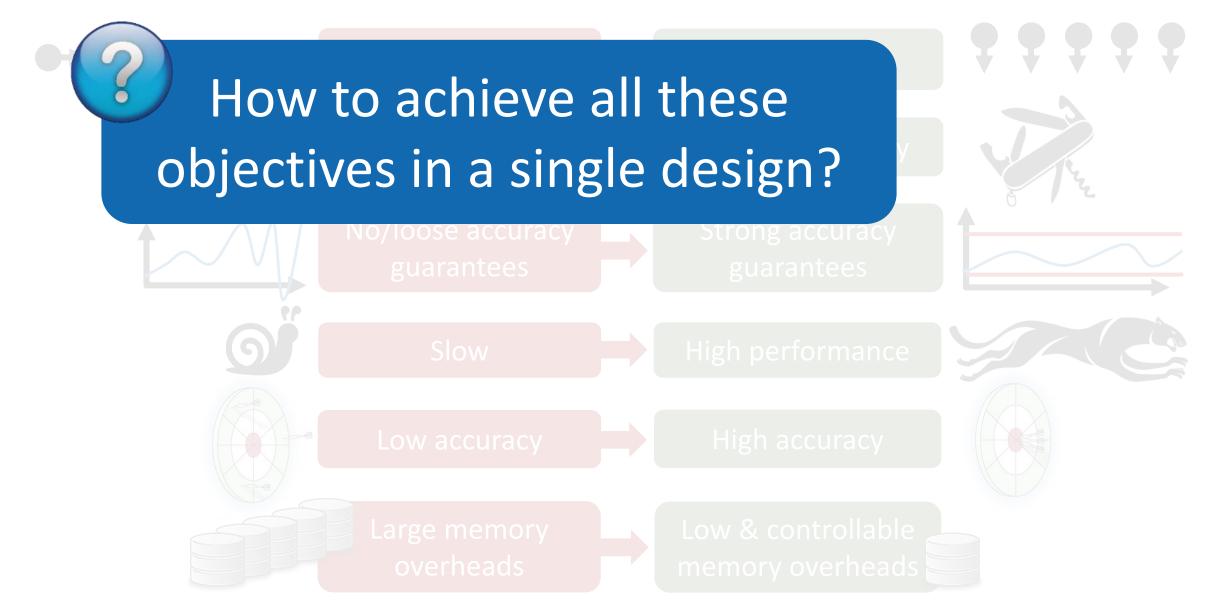
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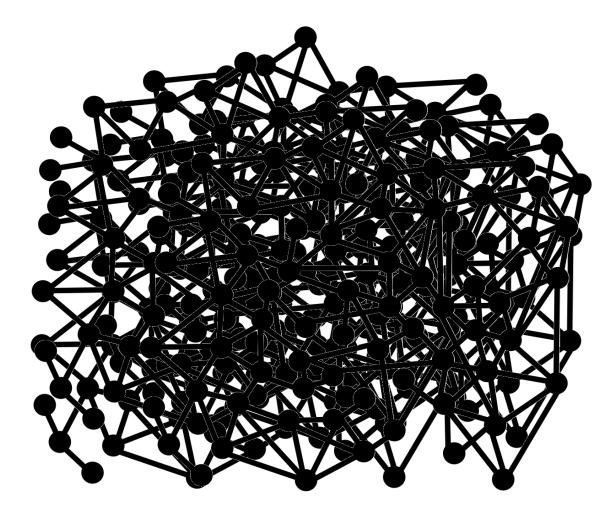
#### **Approximate Graph Processing: Current Issues & Our Objectives**



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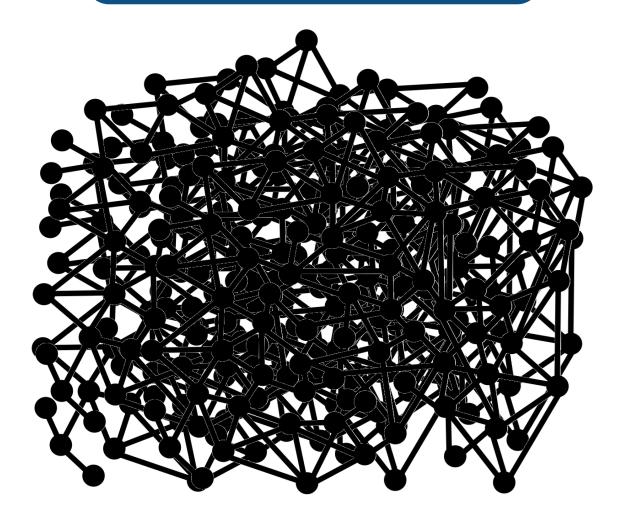






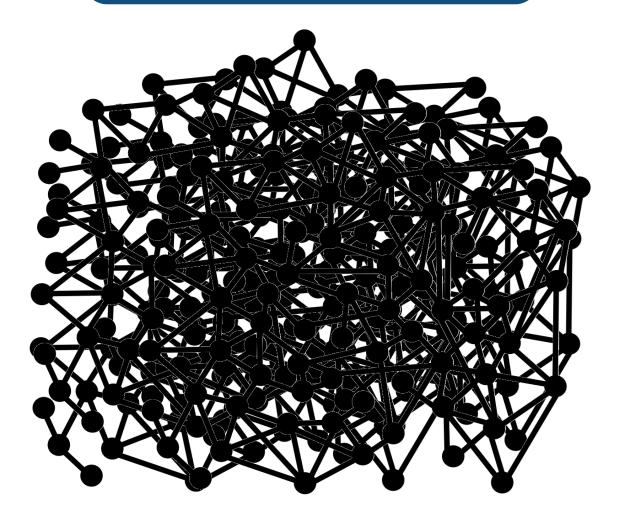


## Keep the original graph

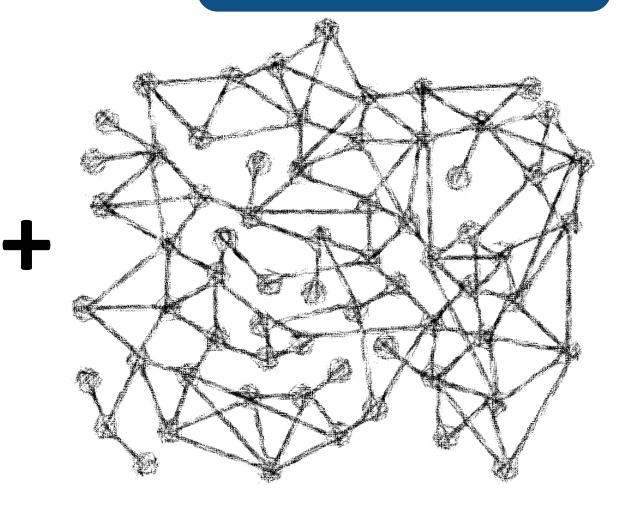




## Keep the original graph



### Maintain a very small "sketch" of a graph

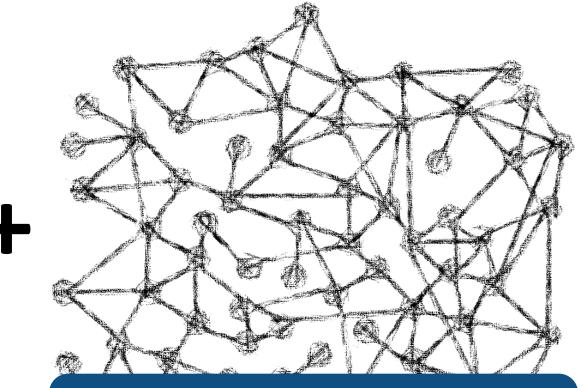




## Keep the original graph



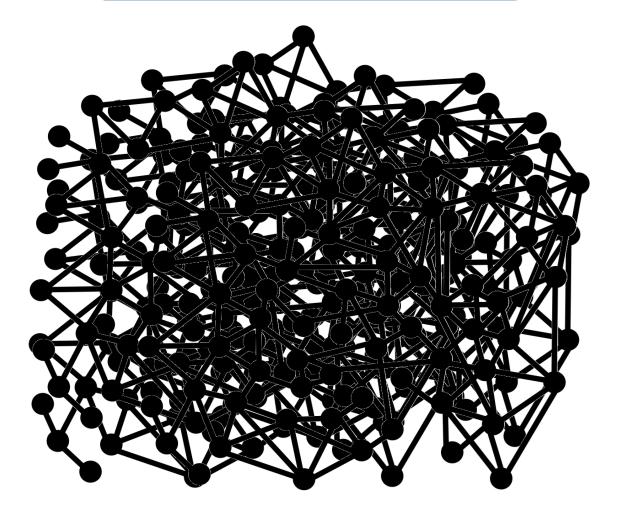
Maintain a very small "sketch" of a graph



Use the sketch to answer performance critical queries



## Keep the original graph



Maintain a very small "sketch" of a graph

What design to use for the sketch, to satisfy all the goals?

Use the sketch to answer performance critical queries





## <u>ProbGraph key idea</u>: Use probabilistic set representations (set sketches) $-\frac{1}{2}$

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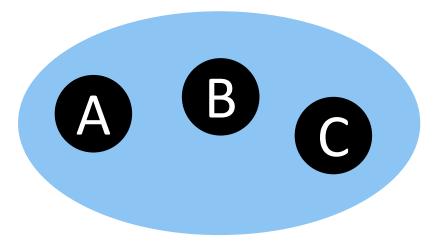


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**<u>ProbGraph key idea</u>**: Use probabilistic set representations (set sketches)



## A set = {A, B, C}





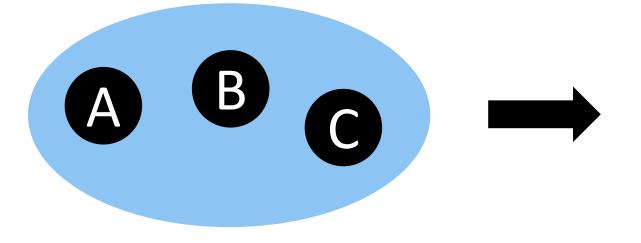




## **ProbGraph key idea:** Use probabilistic set representations (set sketches)

and the manufacture way

A set =  $\{A, B, C\}$ 



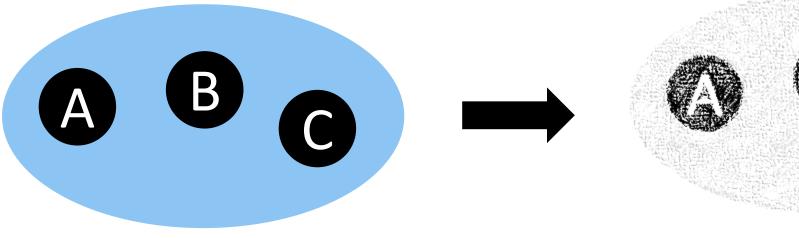


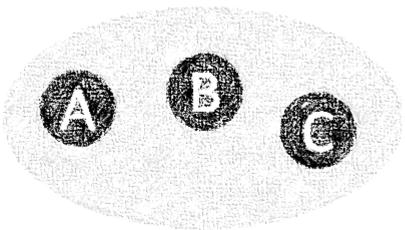




#### **<u>ProbGraph key idea</u>**: Use probabilistic set representations (set sketches)

A set =  $\{A, B, C\}$ 



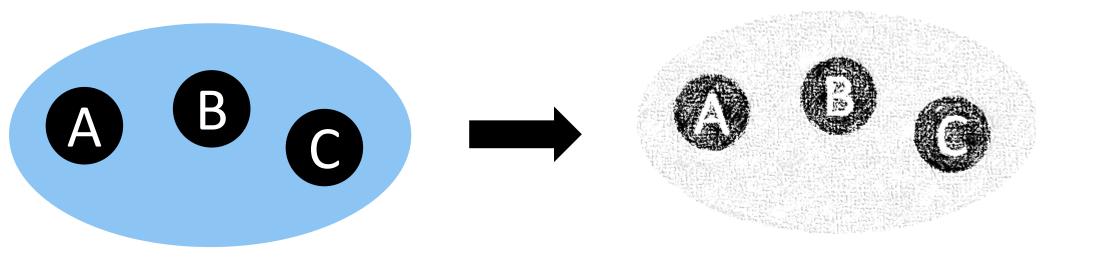






#### **<u>ProbGraph key idea</u>**: Use probabilistic set representations (set sketches)

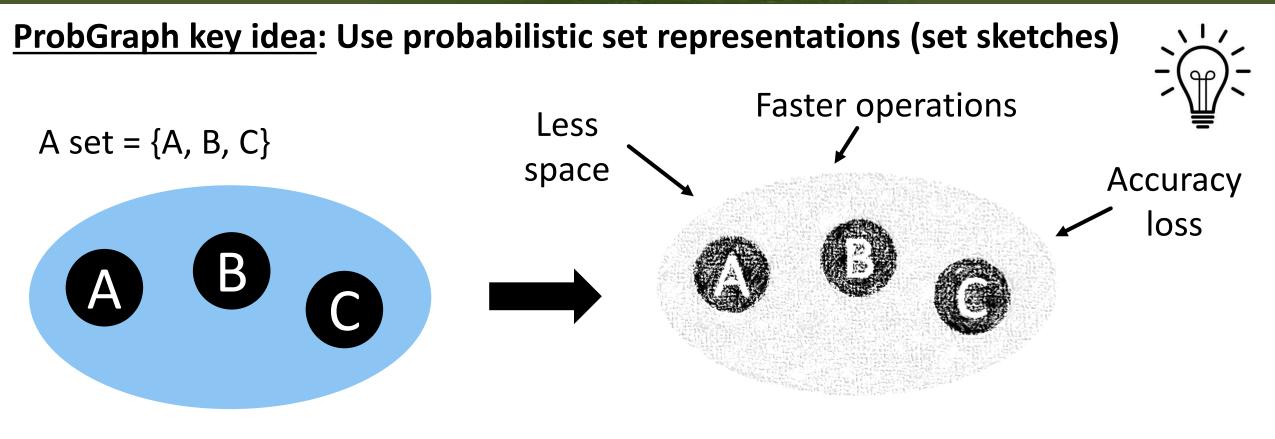
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[1] B. H. Bloom, "Space/time trade-offs in hash coding with allowable errors", CACM, 1970.
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[3] Z. Bar-Yossef et al., "Counting distinct elements in a data stream", in RANDOM, 2002.

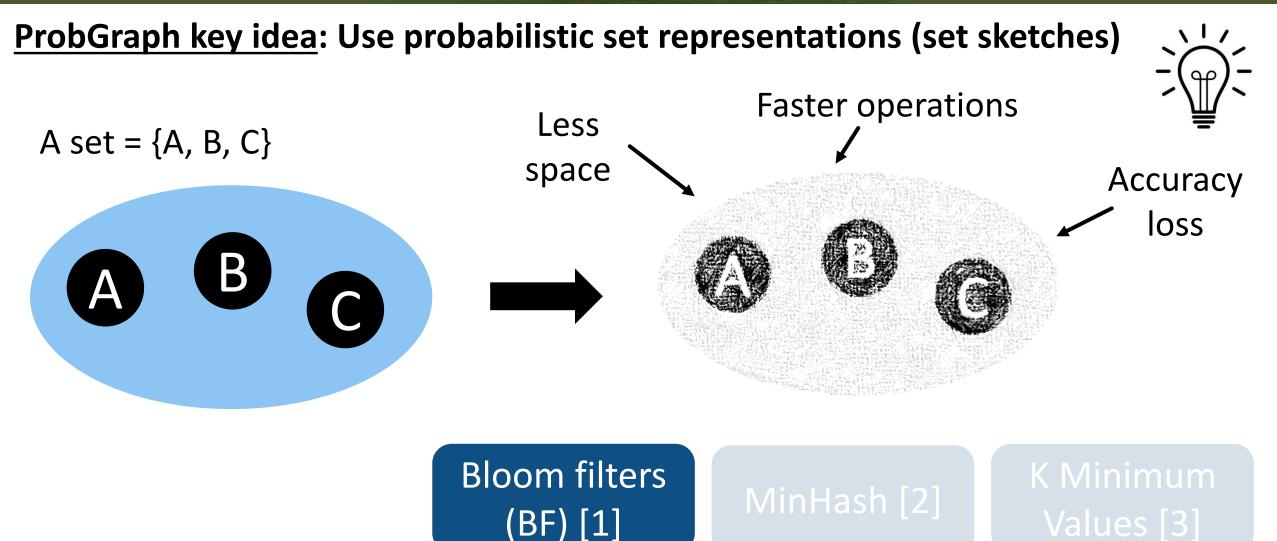






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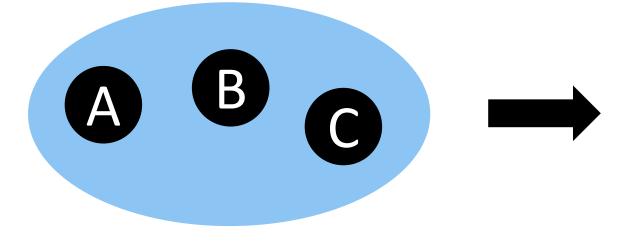




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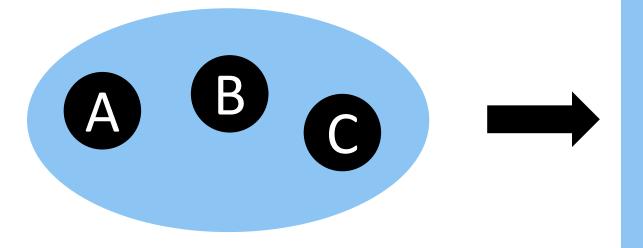
#### **Bloom Filters for Graph Mining**

A set =  $\{A, B, C\}$ 





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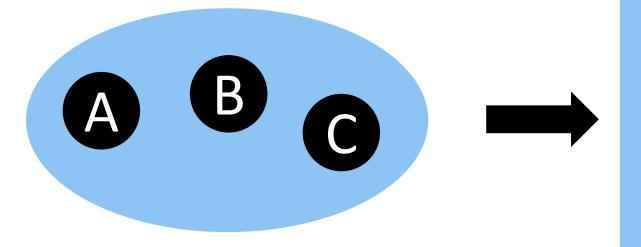
#### Bloom filter $\mathcal{B}_X$ of X

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#### Bitvector of size B<sub>X</sub> [bits]

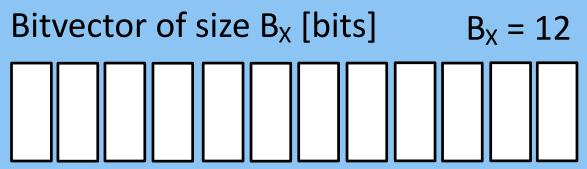


A set =  $\{A, B, C\}$ 



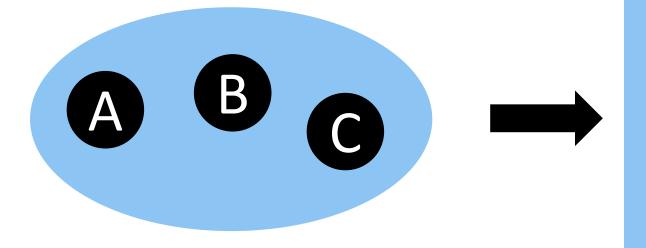
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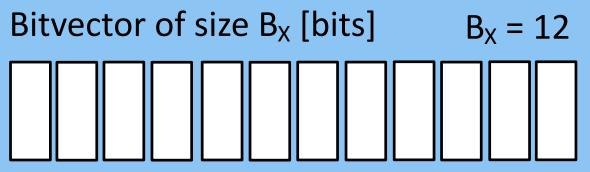


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Bloom filter  $\mathcal{B}_X$  of X

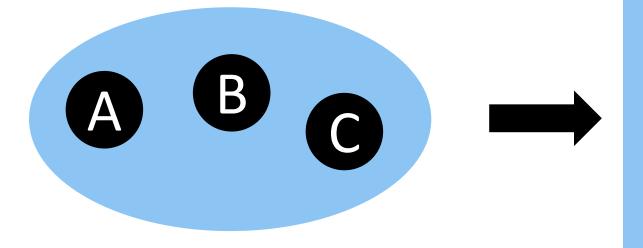
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Hash functions  $h_1, ..., h_b$  $h_i : X \rightarrow \{1, ..., B_X\}$ 

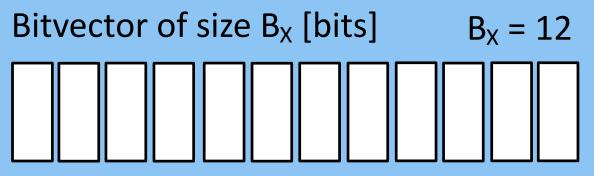


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Bloom filter  $\mathcal{B}_X$  of X

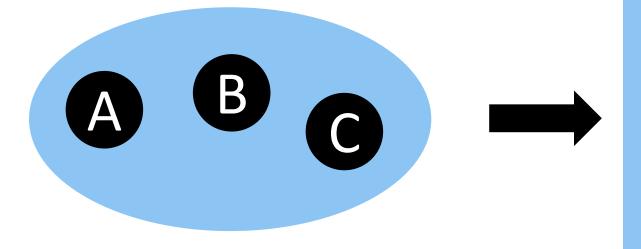
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Hash functions  $h_1, ..., h_b$  $h_i : X \to \{1, ..., B_X\}$ b = 2 $h_2, h_1 : X \to \{1, ..., 12\}$ 



A set =  $\{A, B, C\}$ 

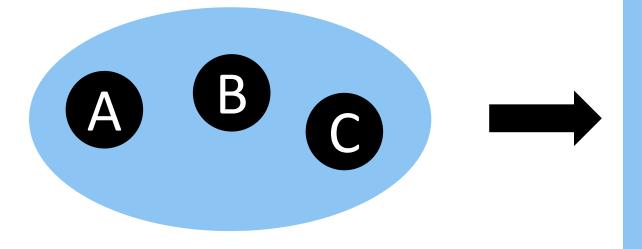


Bloom filter  $\mathcal{B}_{X}$  of X Bitvector of size B<sub>x</sub> [bits]  $B_{\rm X} = 12$ Hash functions  $h_1, ..., h_b$  $h_i: X \rightarrow \{1, ..., B_X\}$ b = 2 $h_2, h_1 : X \rightarrow \{1, ..., 12\}$  $h_1(A) = 3$  $h_2(A) = 5$ 

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A set =  $\{A, B, C\}$ 

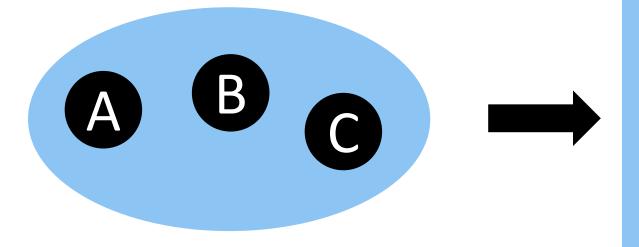


Bloom filter  $\mathcal{B}_{X}$  of X Bitvector of size B<sub>x</sub> [bits]  $B_{\rm X} = 12$ Hash functions  $h_1, ..., h_b$  $h_i: X \rightarrow \{1, ..., B_X\}$ b = 2 $h_2, h_1 : X \rightarrow \{1, ..., 12\}$  $h_1(A) = 3$  $h_2(A) = 5$ = 5

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A set =  $\{A, B, C\}$ 

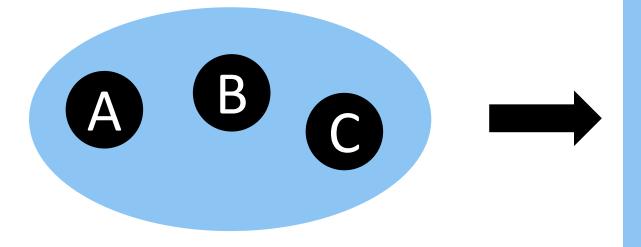


Bloom filter  $\mathcal{B}_{X}$  of X Bitvector of size B<sub>x</sub> [bits]  $B_{\rm X} = 12$ Hash functions  $h_1, ..., h_b$  $h_i: X \rightarrow \{1, ..., B_X\}$ b = 2  $h_2, h_1 : X \rightarrow \{1, ..., 12\}$  $h_1(A) = 3$   $h_1(B) = 1$  $h_2(B) = 5 h_2(B) = 8$ 

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A set =  $\{A, B, C\}$ 

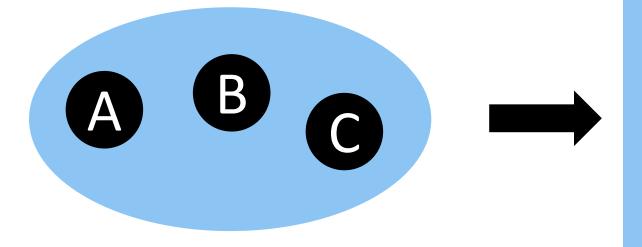


Bloom filter  $\mathcal{B}_{X}$  of X Bitvector of size B<sub>x</sub> [bits]  $B_{x} = 12$ Hash functions  $h_1, ..., h_b$  $h_i: X \rightarrow \{1, ..., B_X\}$ b = 2  $h_2, h_1 : X \rightarrow \{1, ..., 12\}$  $h_1(A) = 3$   $h_1(B) = 1$  $h_2(B) = 5 h_2(B) = 8$ 

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A set =  $\{A, B, C\}$ 

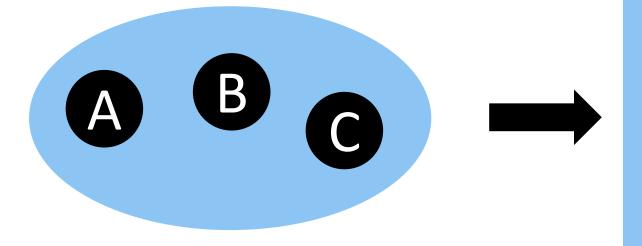


Bloom filter  $\mathcal{B}_{x}$  of X Bitvector of size B<sub>x</sub> [bits]  $B_{x} = 12$ Hash functions  $h_1, ..., h_b$  $h_i: X \rightarrow \{1, ..., B_X\}$ b = 2 $h_2, h_1 : X \rightarrow \{1, ..., 12\}$  $h_1(A) = 3$   $h_1(B) = 1$   $h_1(C) = 4$  $h_2(B) = 5$   $h_2(C) = 11$ 

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A set =  $\{A, B, C\}$ 



Bloom filter  $\mathcal{B}_{x}$  of X Bitvector of size B<sub>x</sub> [bits]  $B_{x} = 12$ Hash functions  $h_1, ..., h_b$  $h_i: X \rightarrow \{1, ..., B_X\}$ b = 2 $h_2, h_1 : X \rightarrow \{1, ..., 12\}$  $h_1(A) = 3$   $h_1(B) = 1$   $h_1(C) = 4$  $h_2(B) = 8 h_2(C) = 11$ 

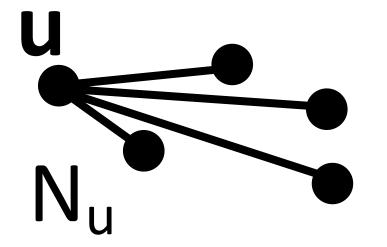
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#### **Bloom Filters for Graph Mining**



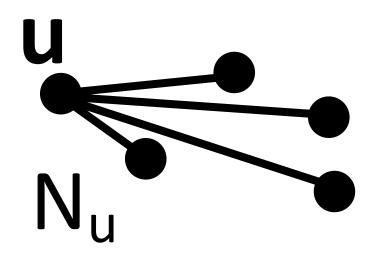




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#### **Bloom Filters for Graph Mining**

# Each neighborhood $N_u$ is a <u>set</u> of vertices



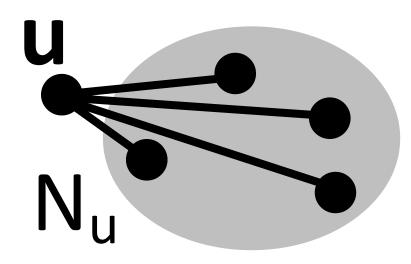




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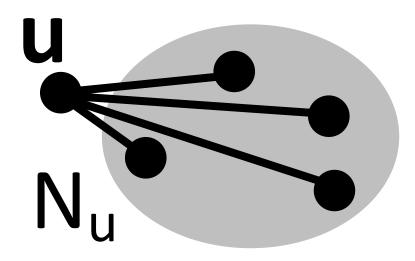




## Each neighborhood N<sub>u</sub> is a <u>set</u> of vertices



## "Sketch" each N<sub>u</sub> with a Bloom filter

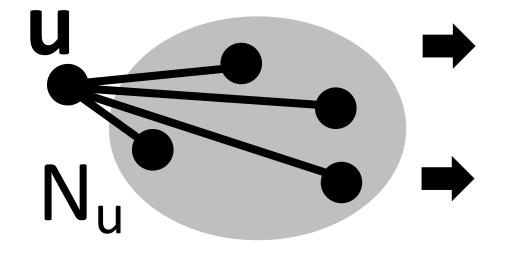




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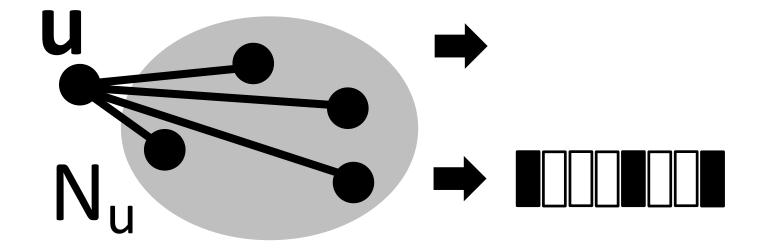


## Each neighborhood N<sub>u</sub> is a <u>set</u> of vertices



## "Sketch" each N<sub>u</sub> with a Bloom filter

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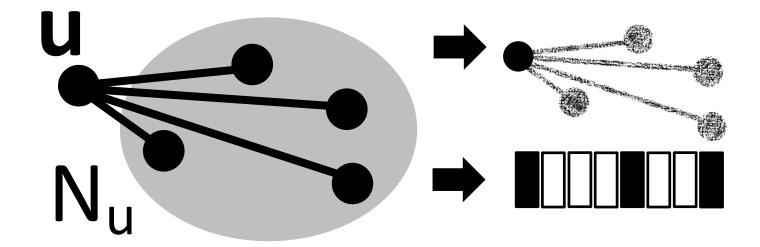




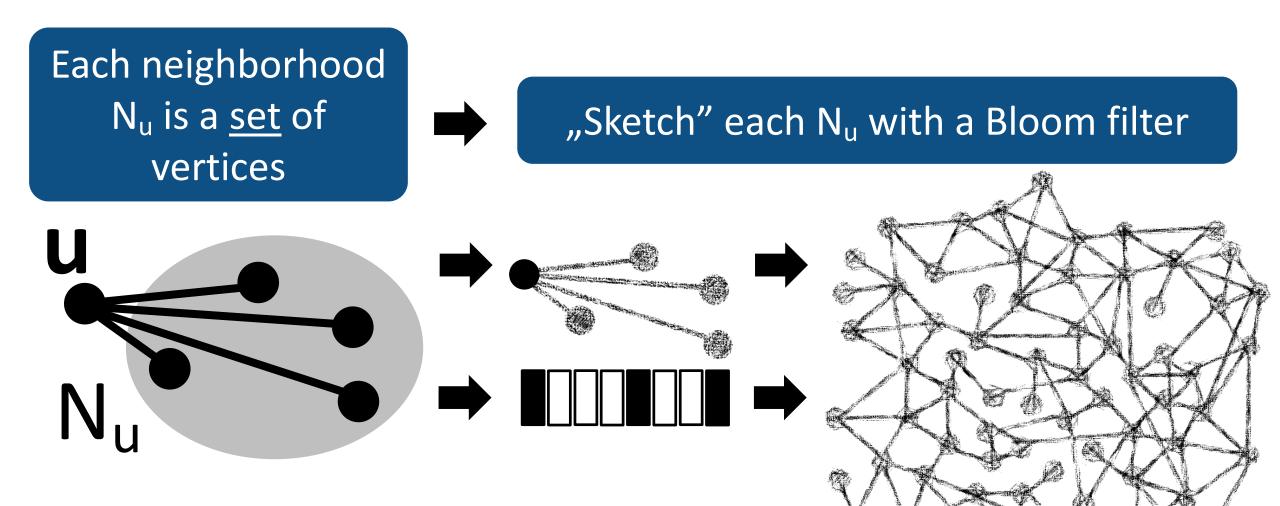
## Each neighborhood N<sub>u</sub> is a <u>set</u> of vertices



## "Sketch" each N<sub>u</sub> with a Bloom filter







11





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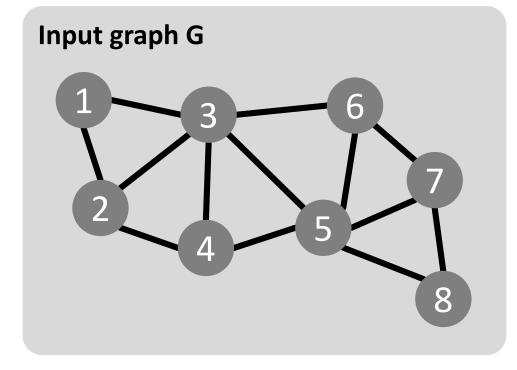
#### **ProbGraph: Summary of Design**





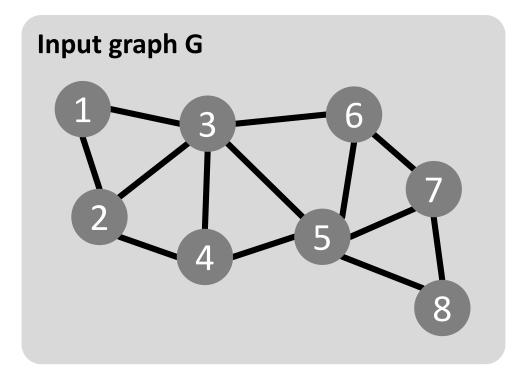
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#### **ProbGraph: Summary of Design**



#### \*\*\*SPCL

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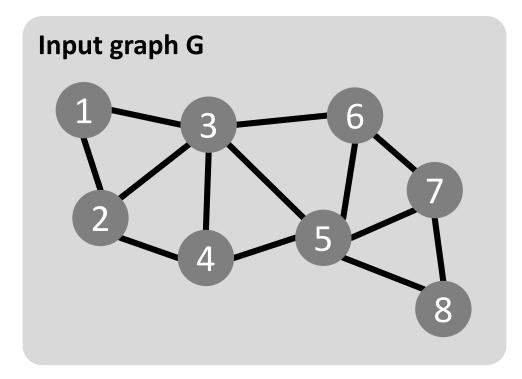


Standard graph representation (e.g., CSR)

12

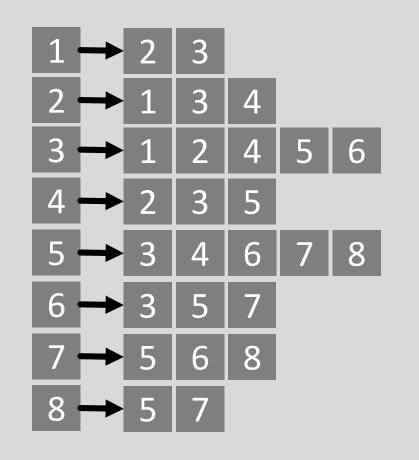
#### \*\*\*SPCL

#### **ProbGraph: Summary of Design**

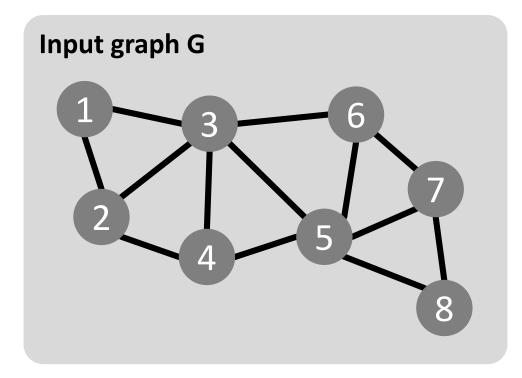


#### Standard graph representation (e.g., CSR)

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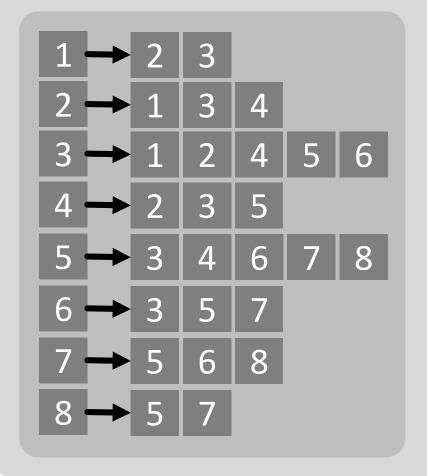


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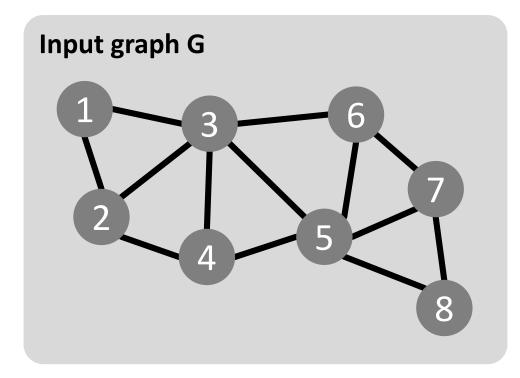


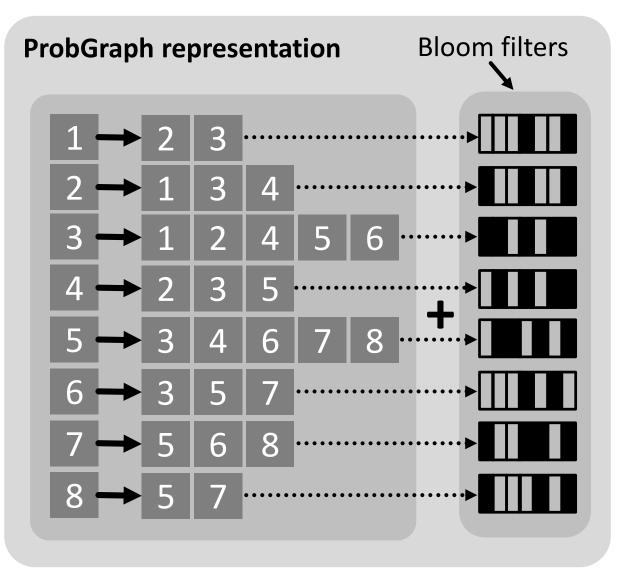
#### **ProbGraph representation**

State - ----



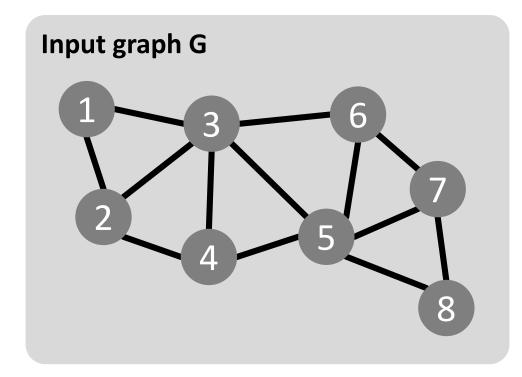
#### **ProbGraph: Summary of Design**

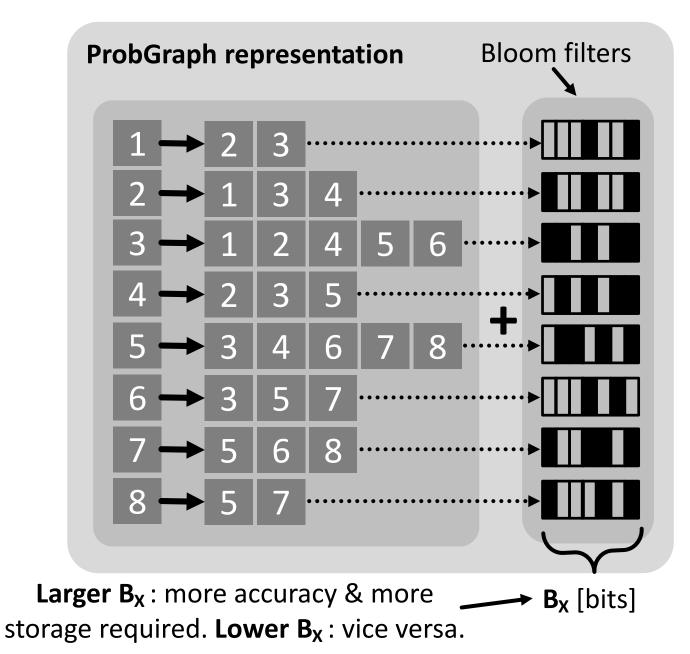




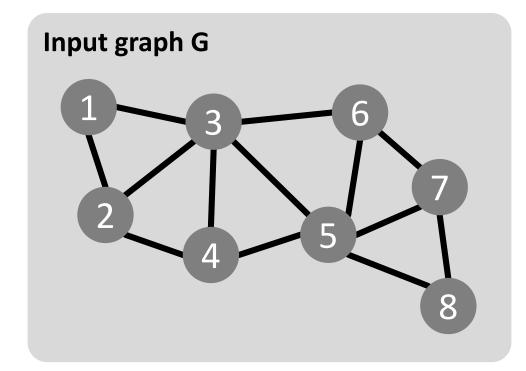
The seal

### ProbGraph: Summary of Design

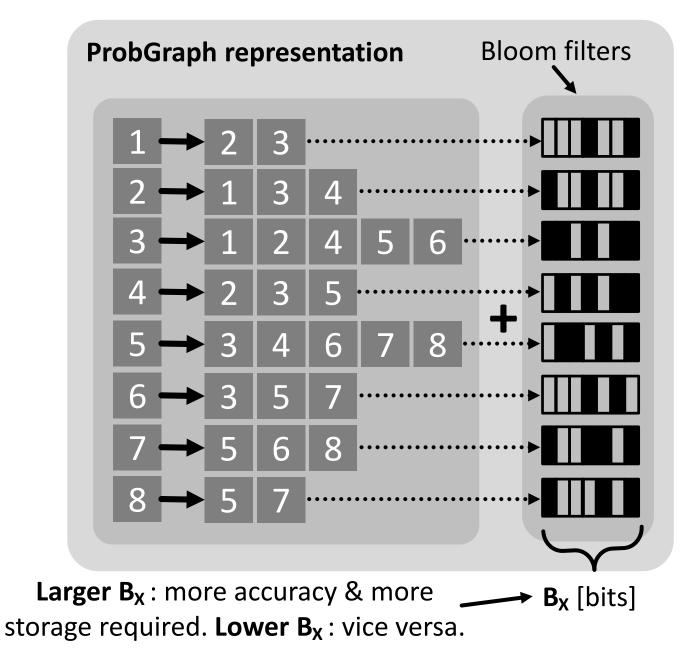




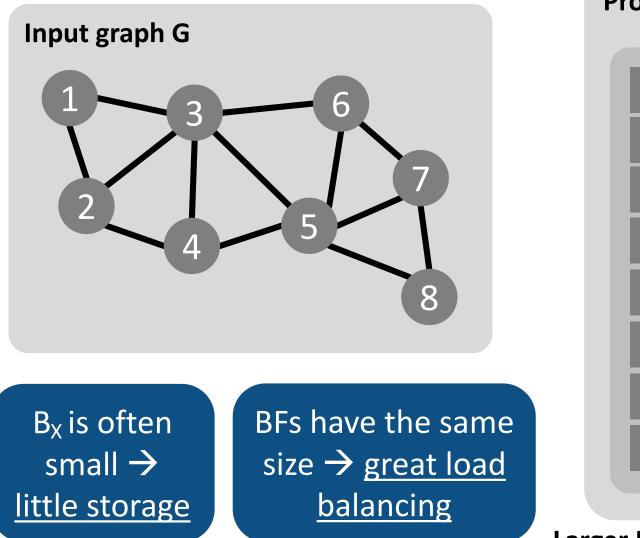
### **ProbGraph: Summary of Design**

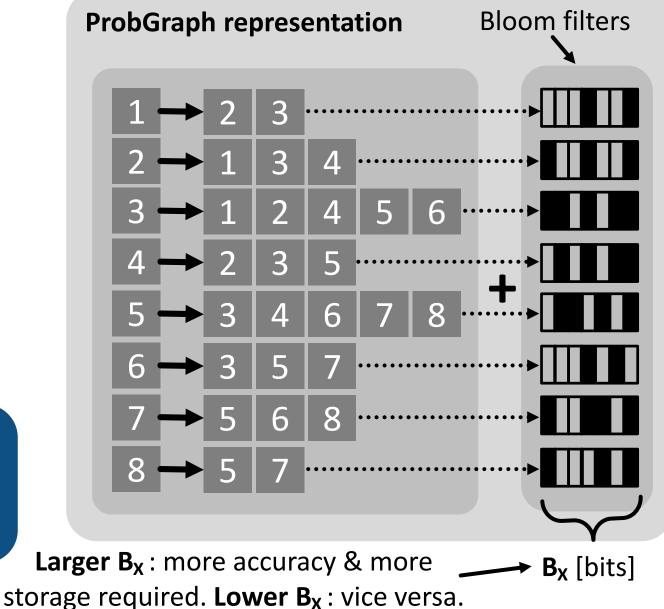


B<sub>x</sub> is often small → <u>little storage</u>



### **ProbGraph: Summary of Design**









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Traditional BF use case: presence tracking





Proventiers

Traditional BF use case: presence tracking









Printer

Traditional BF use case: presence tracking

# **A BF cache** tracking the presence of data







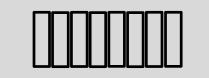


Traditional BF use case: presence tracking



**A BF cache** tracking the presence of data

the second second





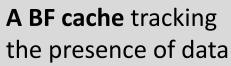


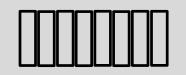


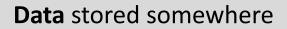


How does our idea compare to other Bloom filter use cases?

Traditional BF use case: presence tracking









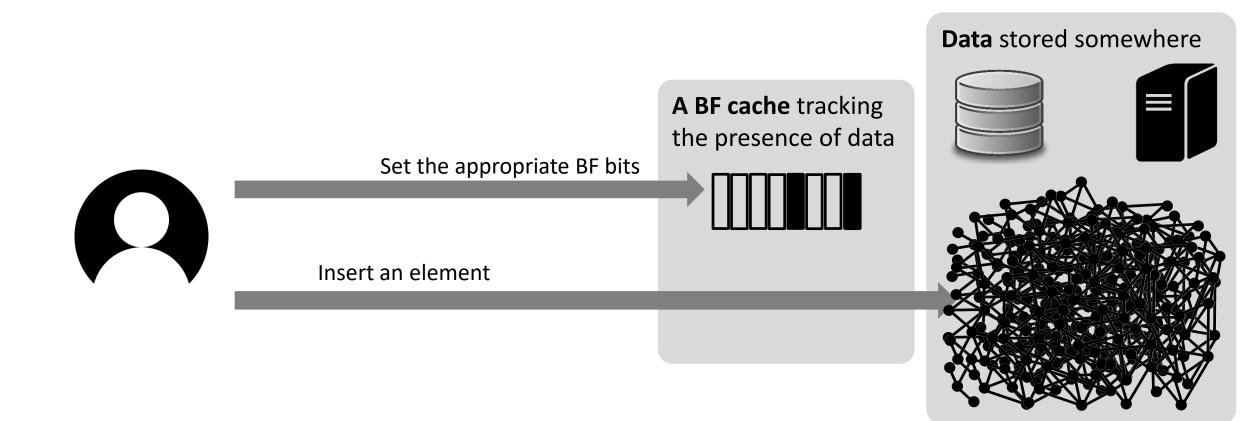


Insert an element





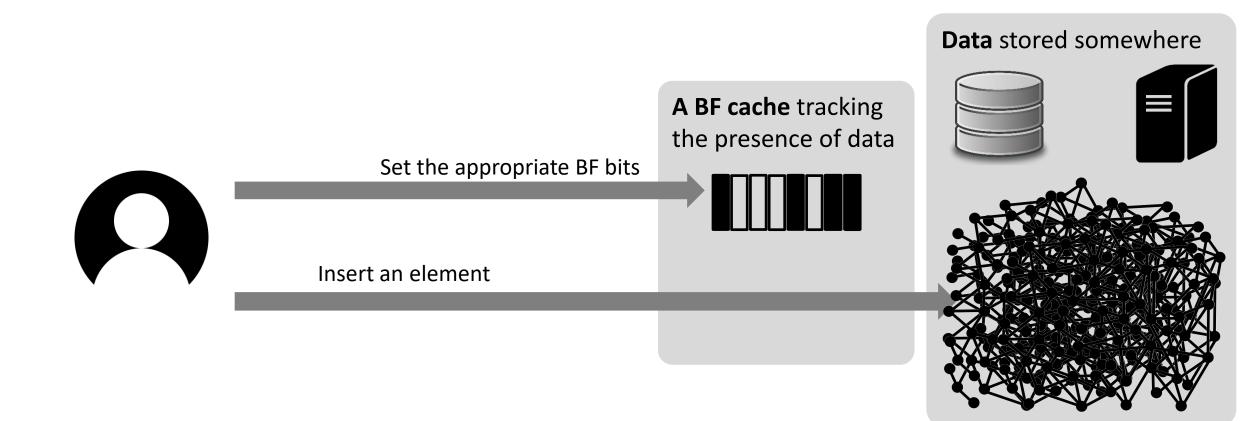
Traditional BF use case: presence tracking







Traditional BF use case: presence tracking



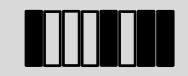


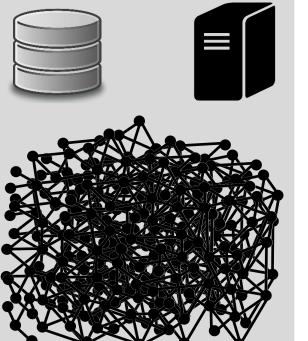


Traditional BF use case: presence tracking



**A BF cache** tracking the presence of data

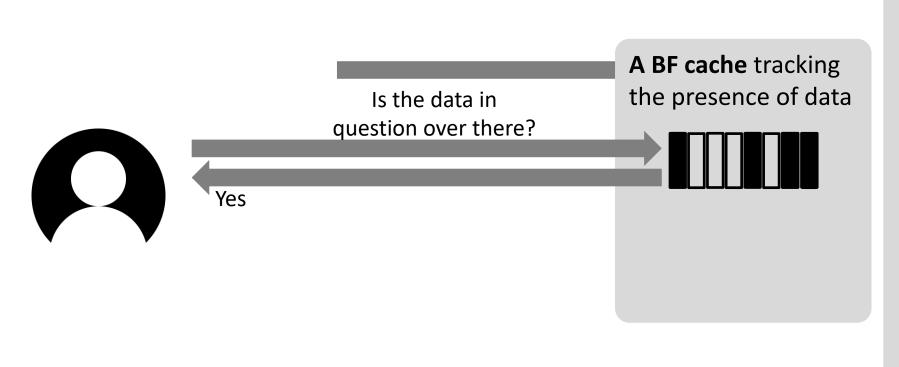








Traditional BF use case: presence tracking

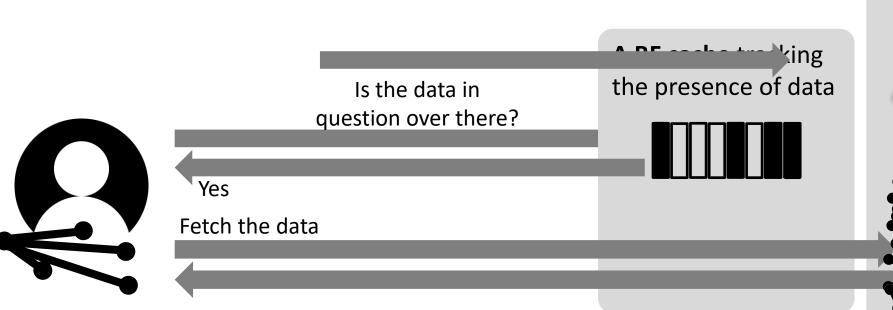




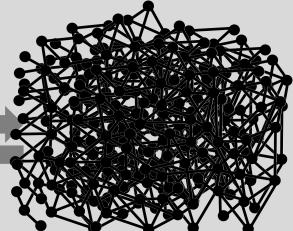




Traditional BF use case: presence tracking



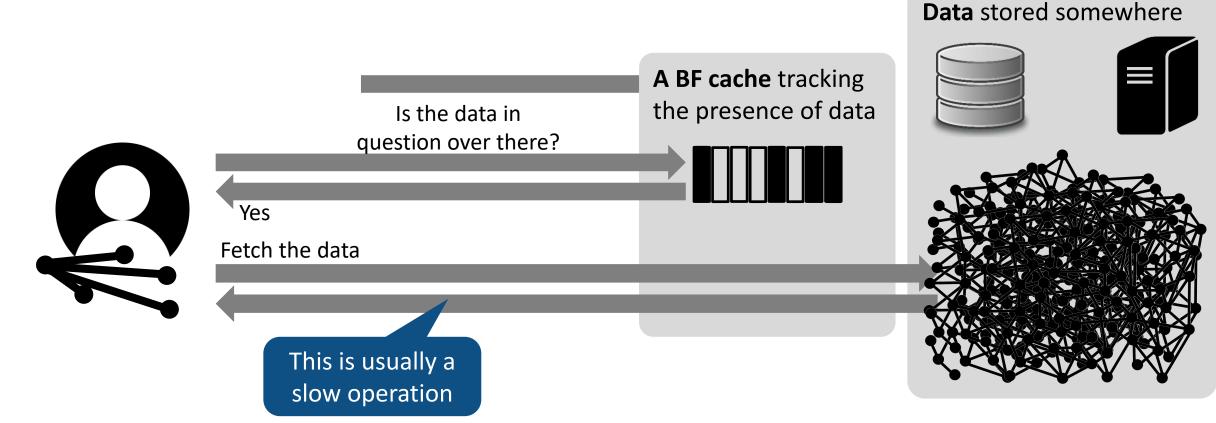








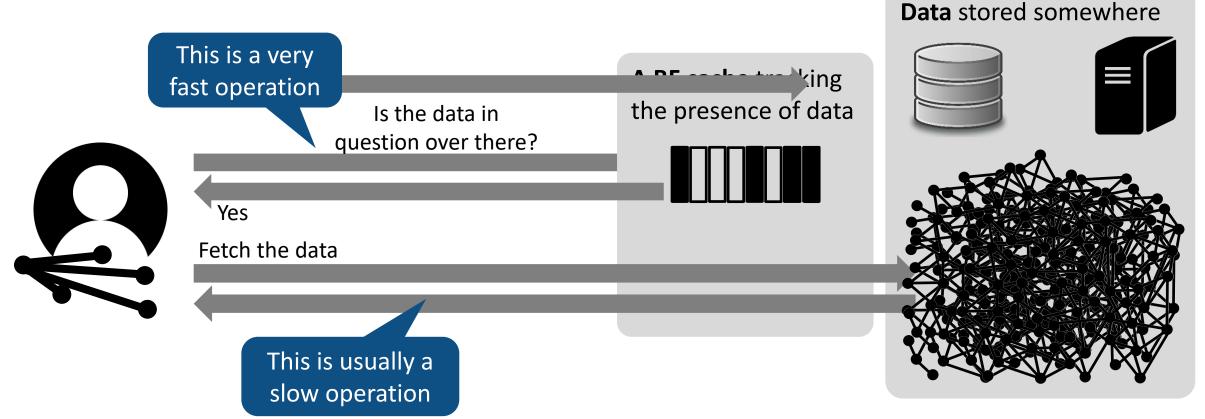
Traditional BF use case: presence tracking







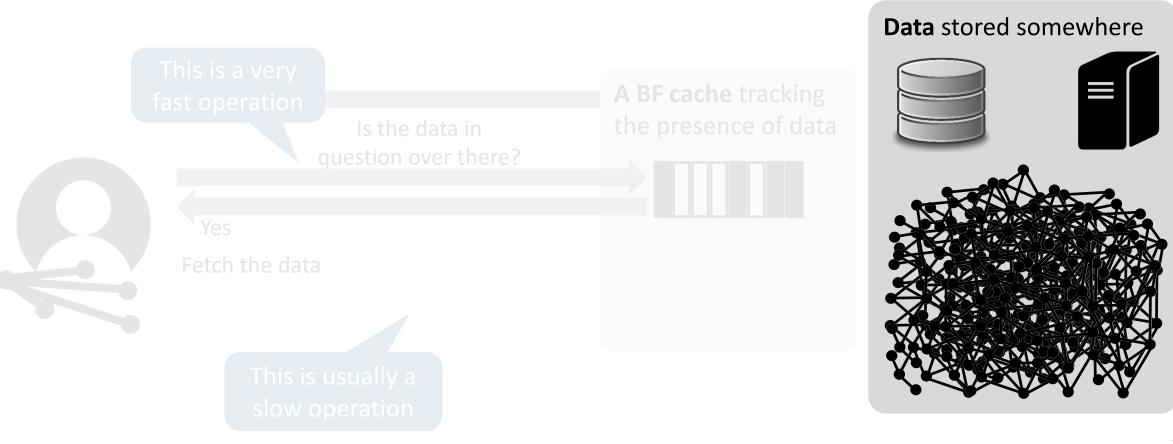
Traditional BF use case: presence tracking







## r Bloom filter use cases?



All Charles and





## r Bloom filter use cases?

# We use BFs as a sketch of the actual dataset

F cools the king e presence of data

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#### Data stored somewhere



Yes

Fetch the data

This is usually a slow operation







## r Bloom filter use cases?

# We use BFs as a sketch of the actual dataset

**BF cache** tracking e presence of data

All the second second second

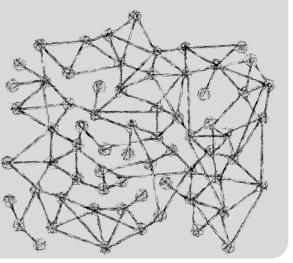
#### Data stored somewhere



Yes

Fetch the data

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## Bloom filter use cases?

We use BFs as a sketch of the actual dataset

e presence of data

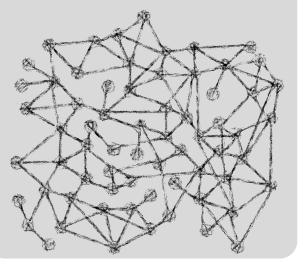
#### Data stored somewhere



Yes

etch the data

How do we <u>exactly</u> use these sketches to benefit graph mining?



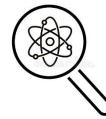


Contra and and





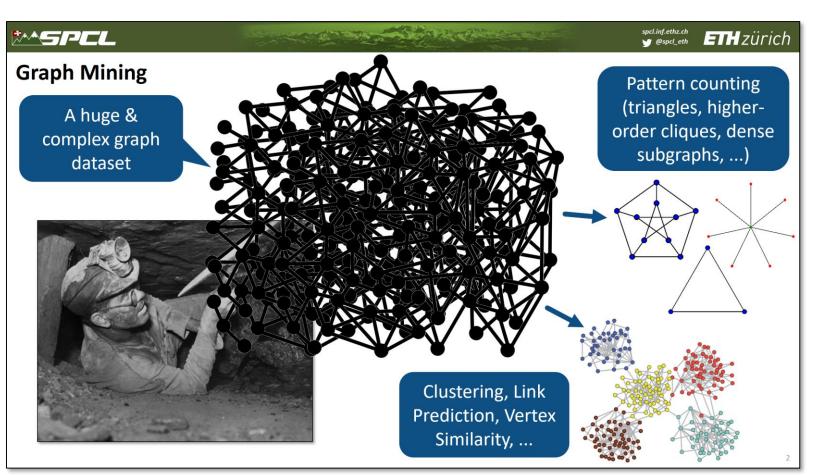
Carlo and and a



 $|X \cap Y|$ 

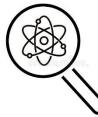


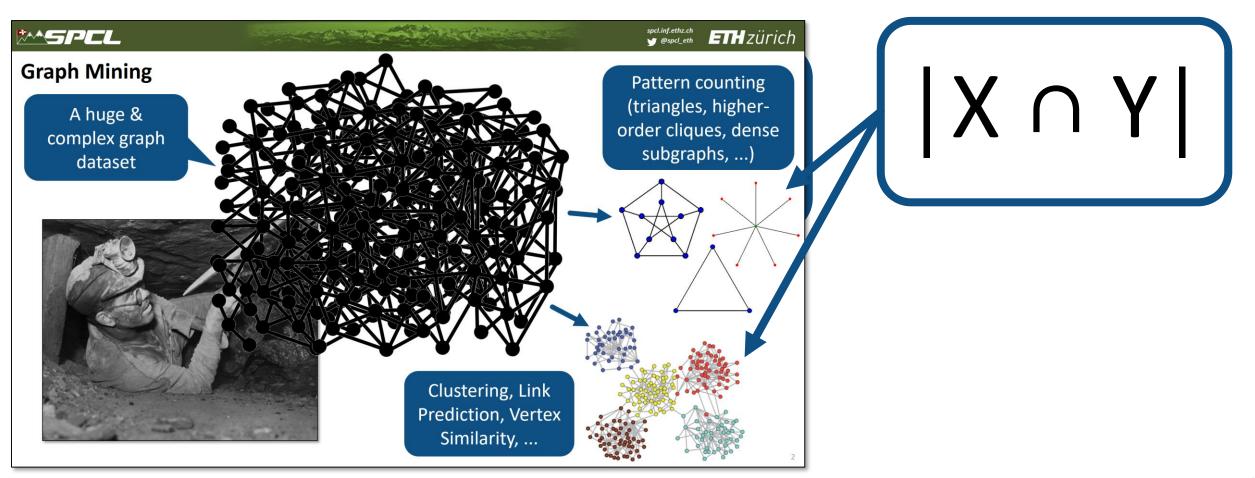




# $|X \cap Y|$

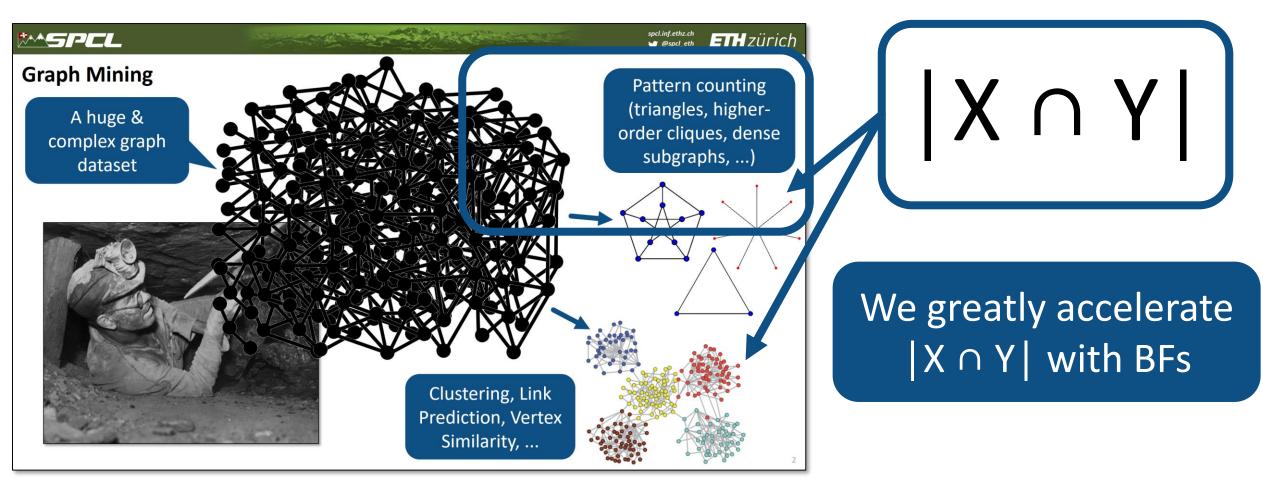
















and the section was

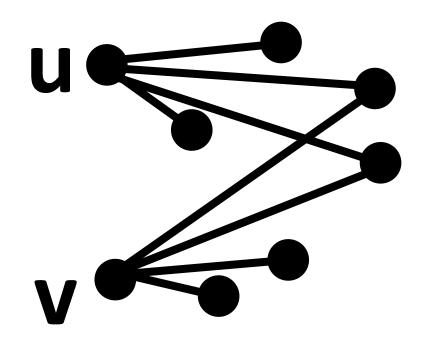


## ProbGraph key idea, continued



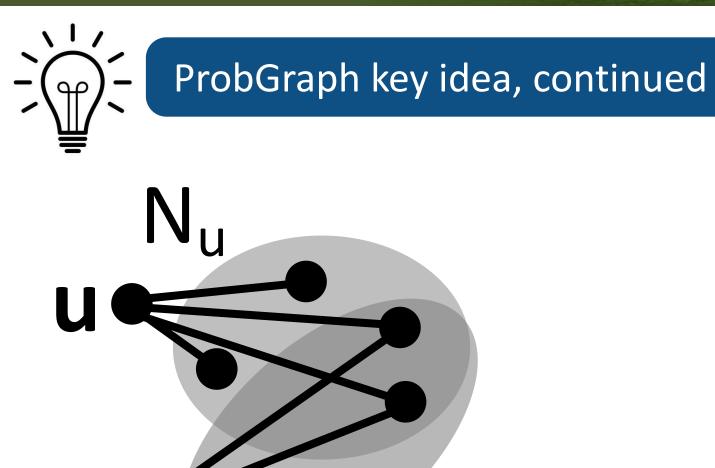
The second







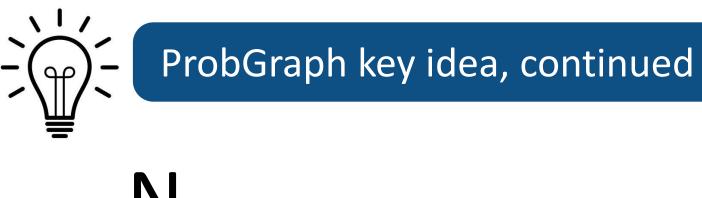
State and some

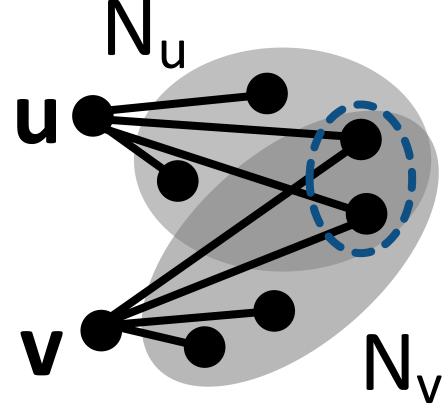


 $N_v$ 



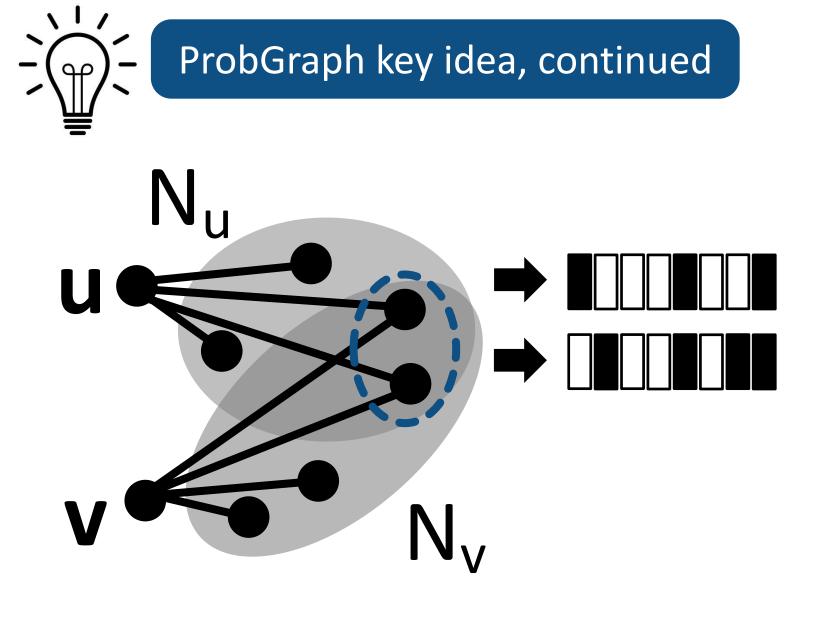
and the second





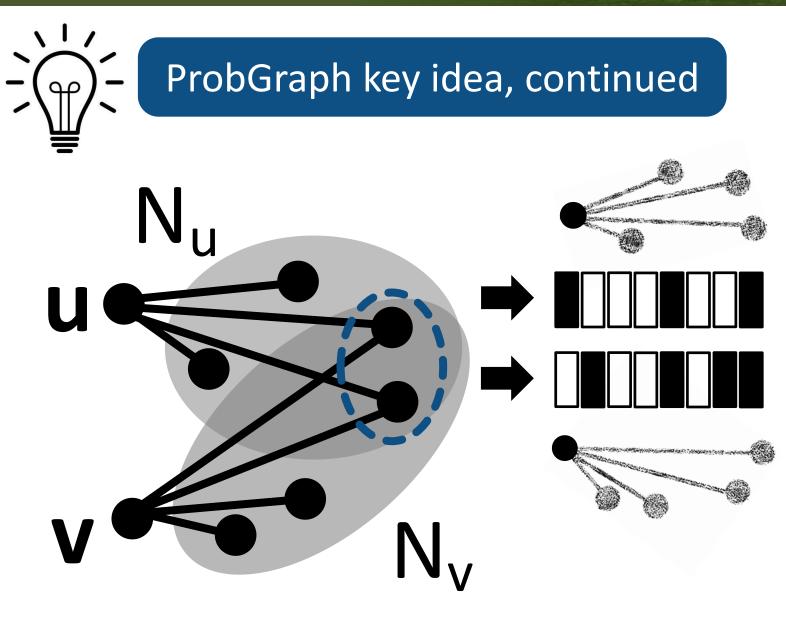


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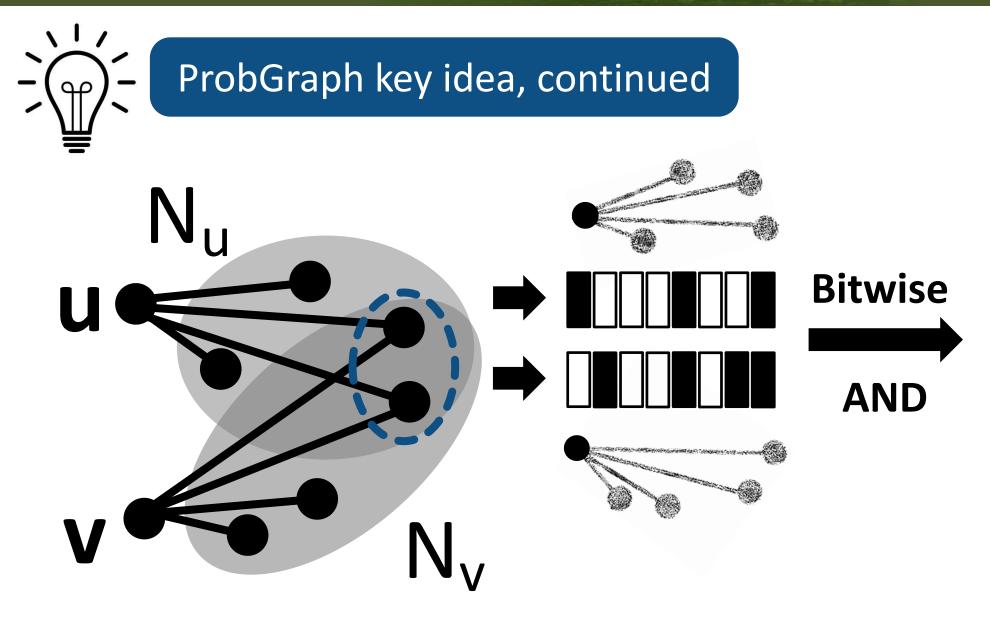




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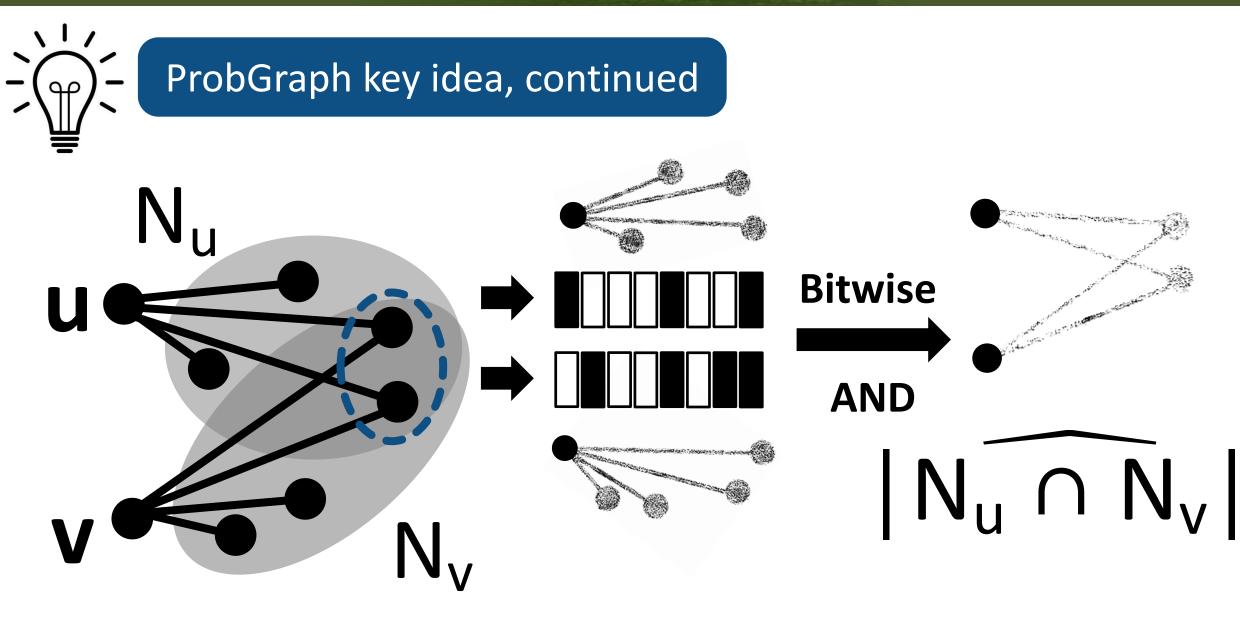


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16

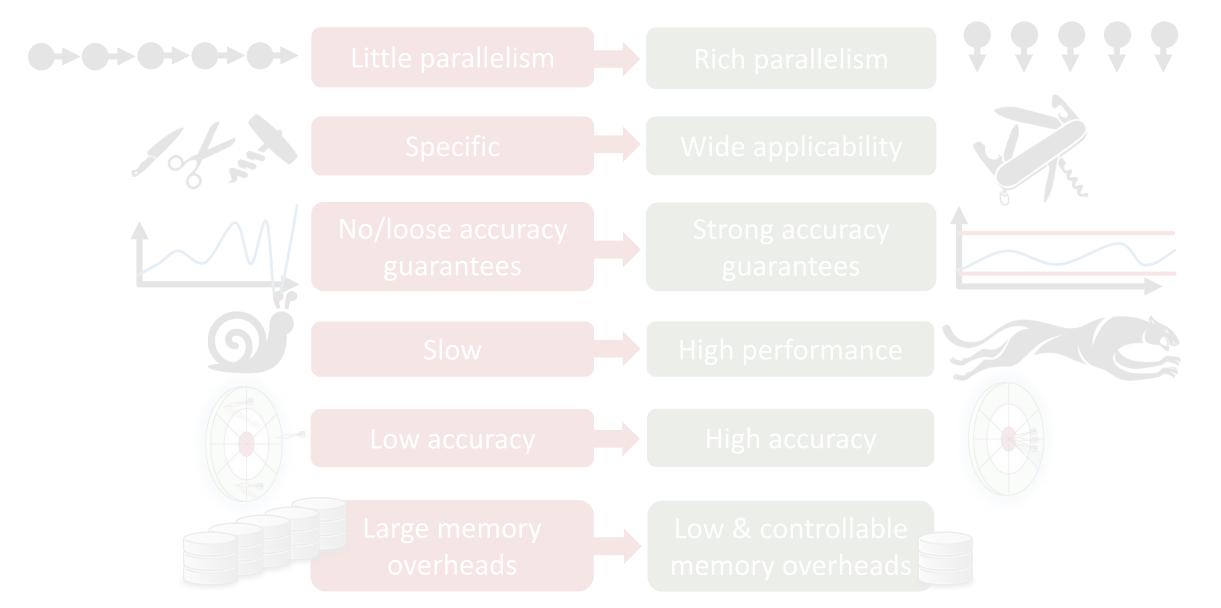


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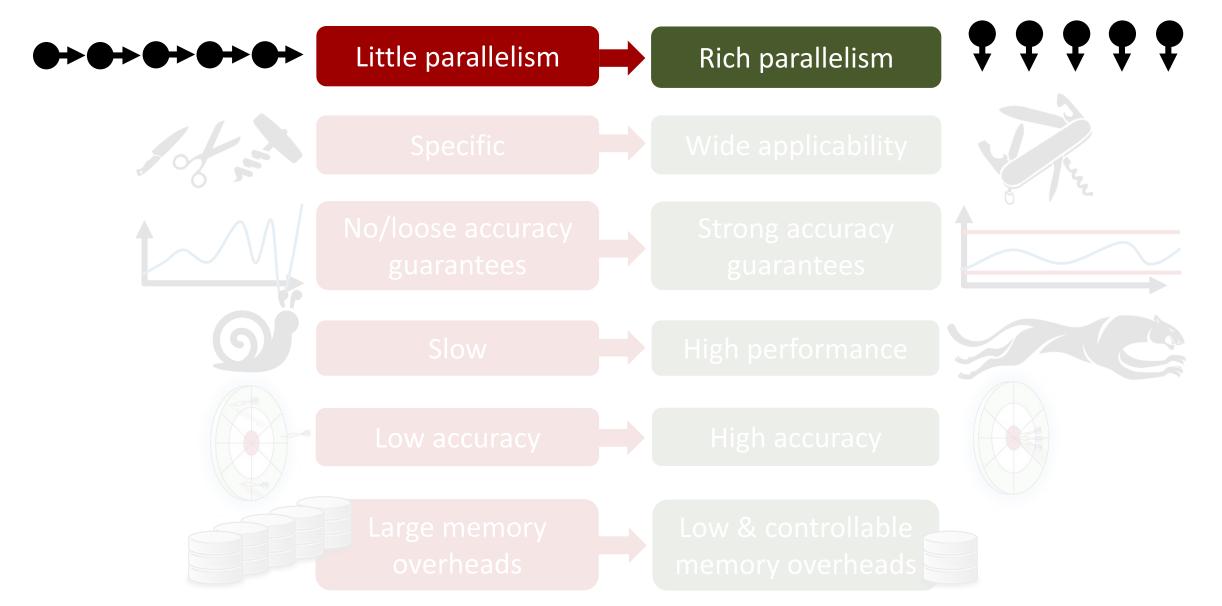


#### **Approximate Graph Processing: Our Objectives**



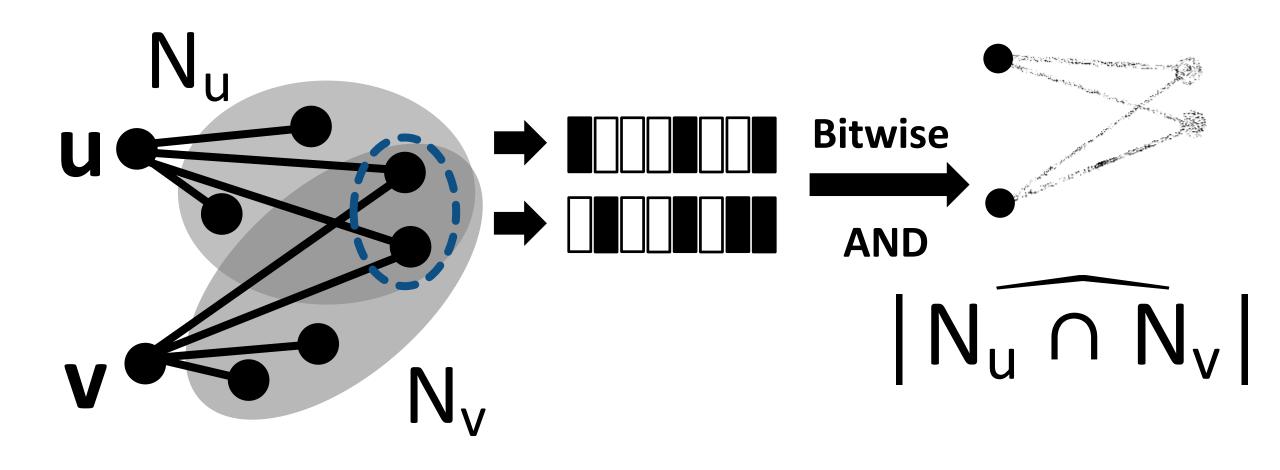
A DESCRIPTION OF THE PARTY OF T







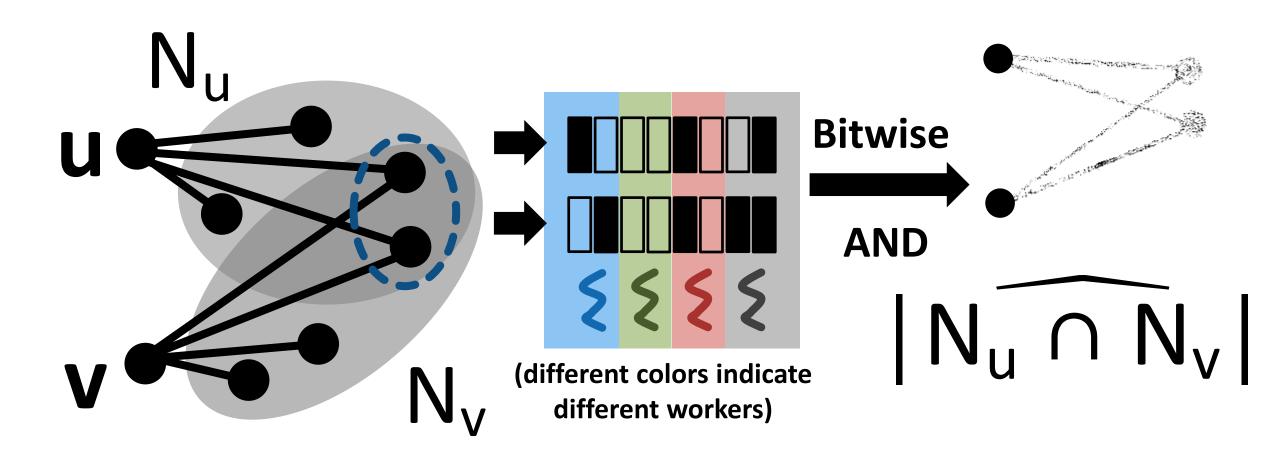
#### **ProbGraph: Fast & Parallel Execution**



State and and



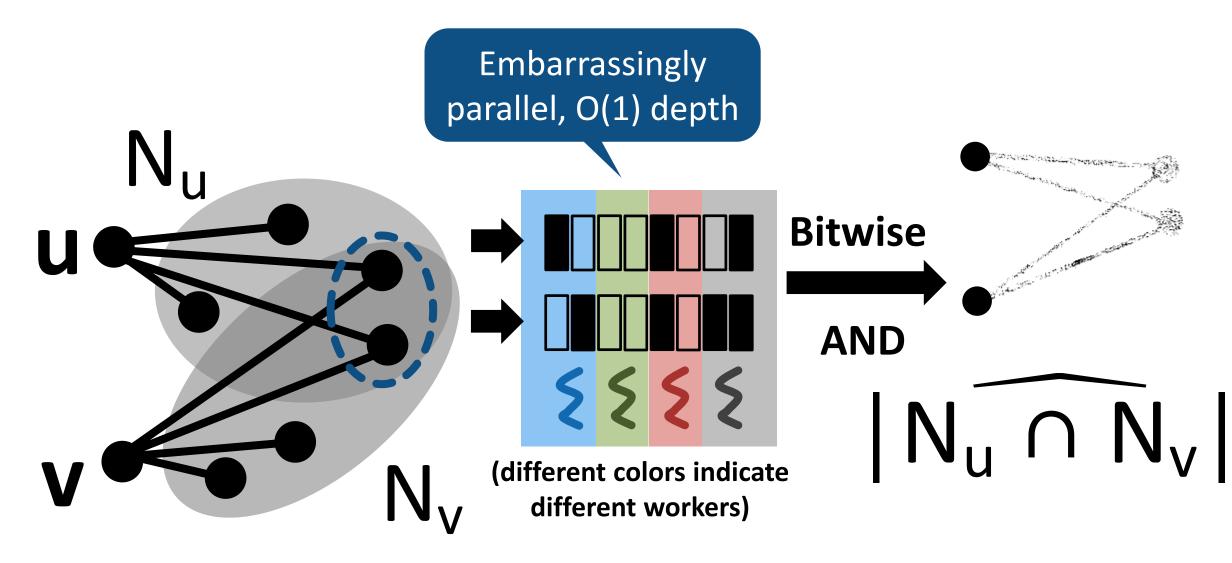
#### **ProbGraph: Fast & Parallel Execution**



and and and

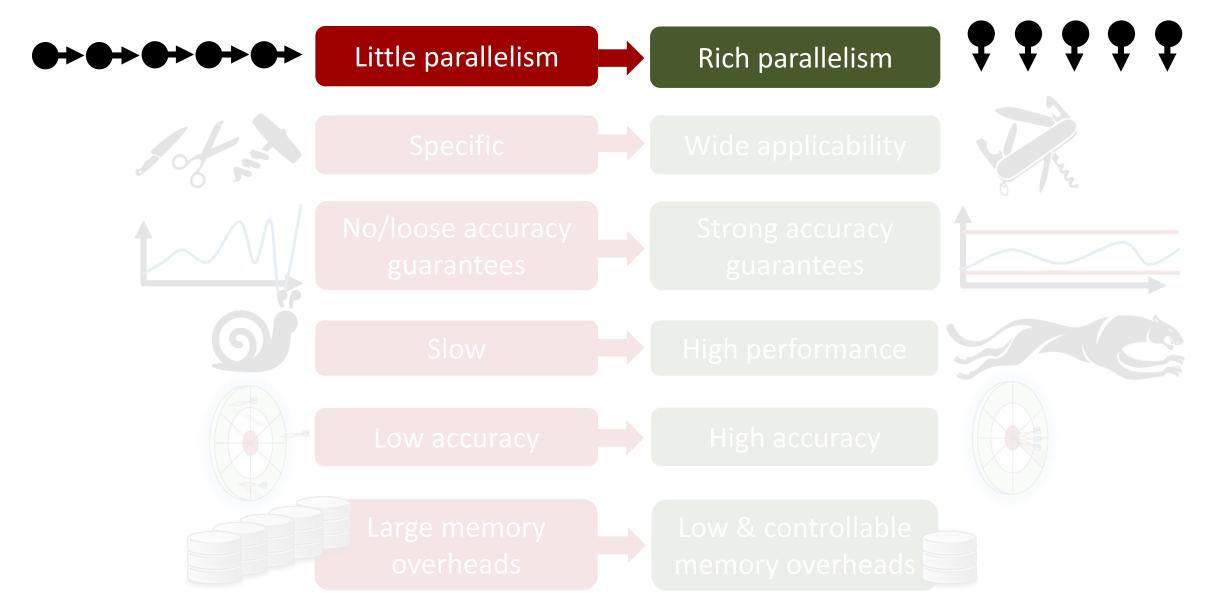


#### **ProbGraph: Fast & Parallel Execution**

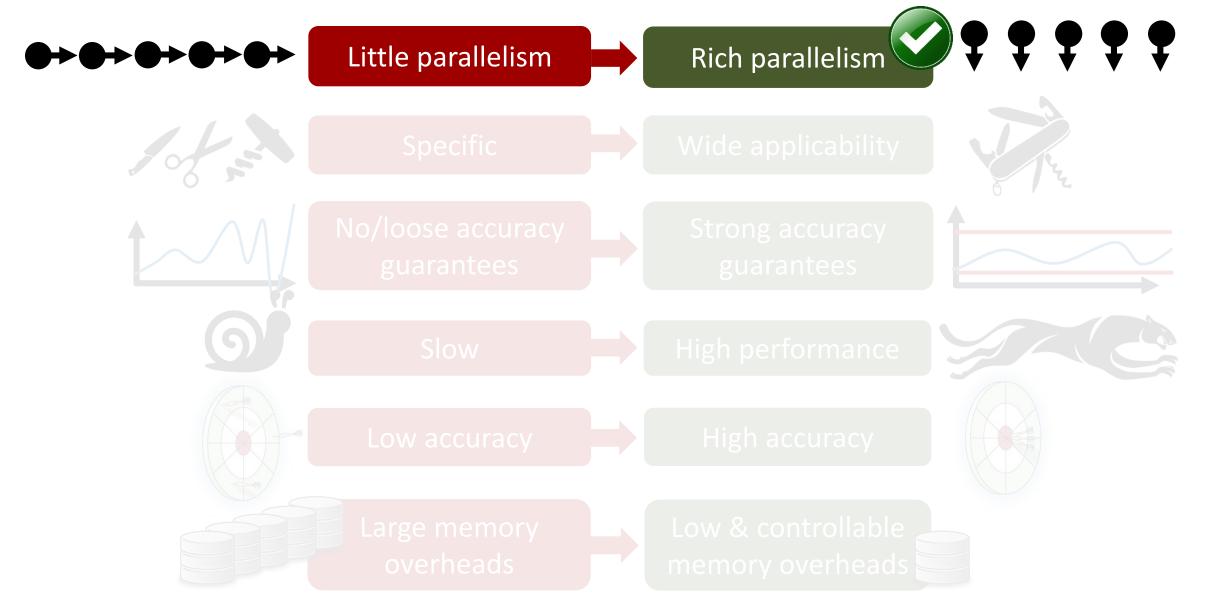


the section



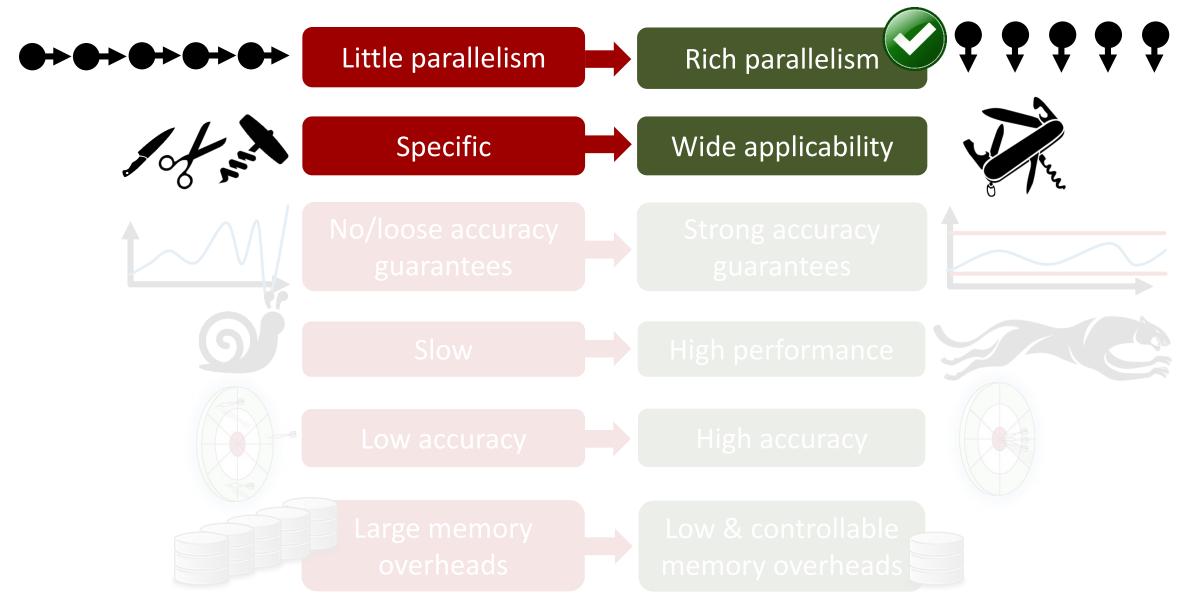




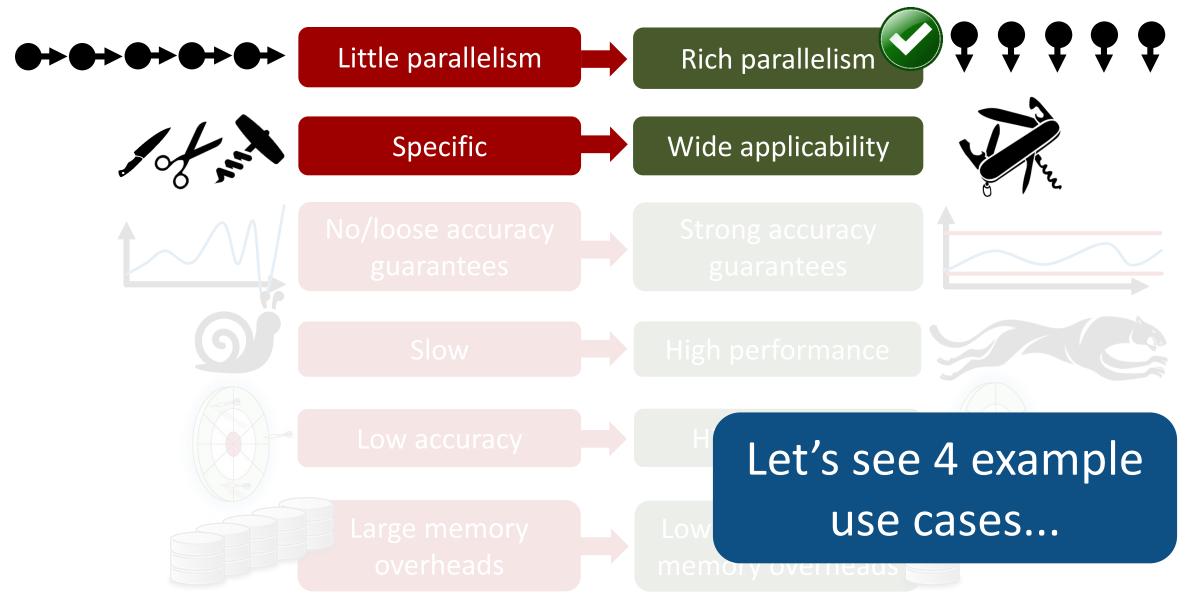


Development and the second









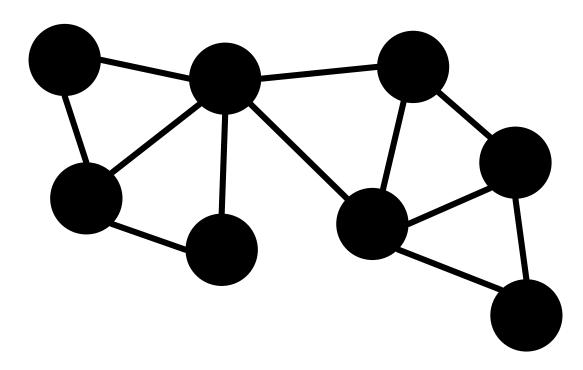
A CALL STREET







# Which links will appear?



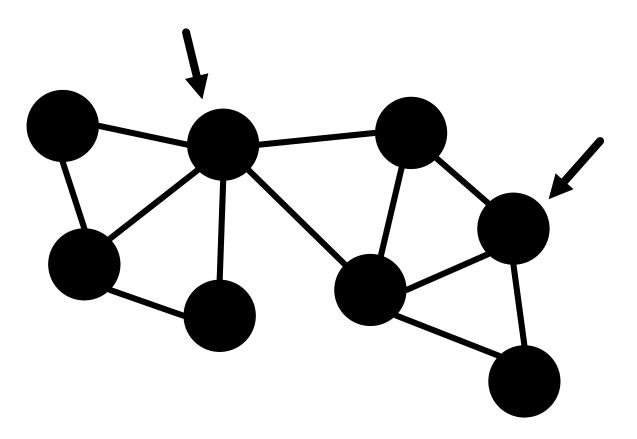
all the second







# Which links will appear?



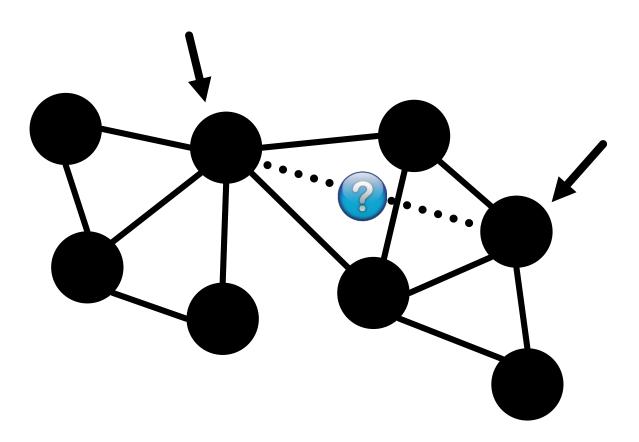
all the second







# Which links will appear?



all the sectors

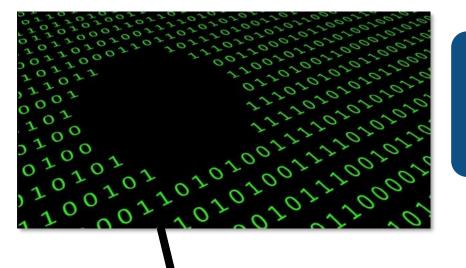


# Which links will appear?

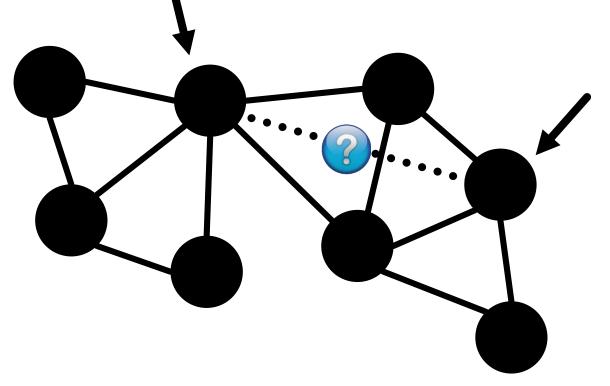


Which links

are missing?

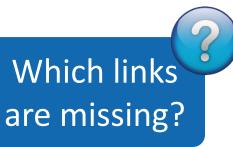








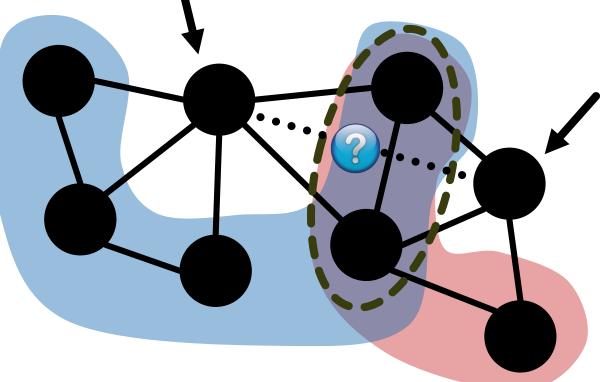
# Which links will appear?







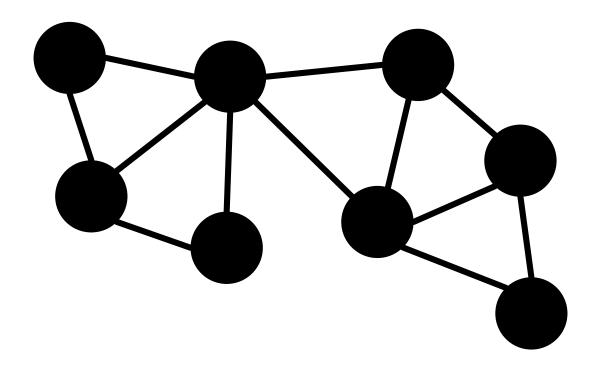








### **Use Case 2: Clique Counting**

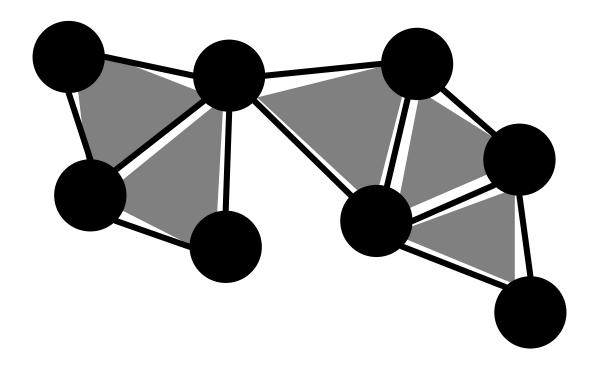


and and and and and





### **Use Case 2: Clique Counting**



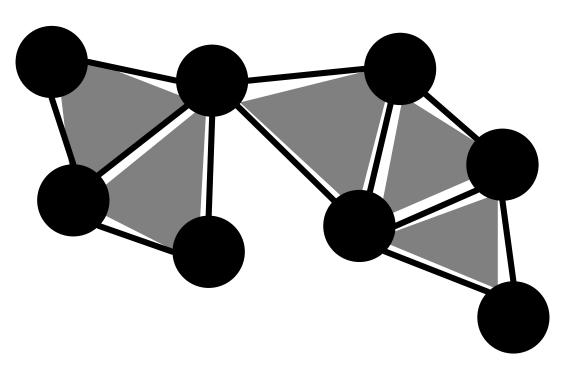
a later and a second





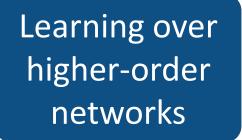
### **Use Case 2: Clique Counting**

a start and and

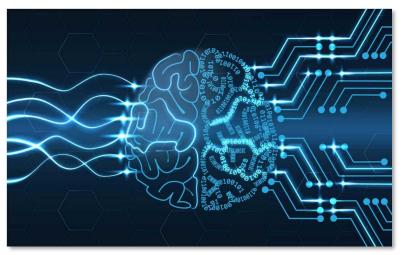


#### \*\*\*SPCL

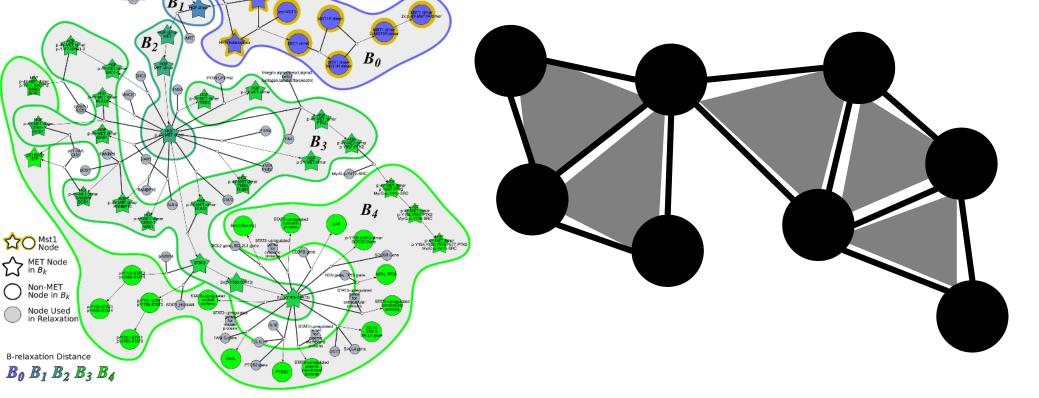
### **Use Case 2: Clique Counting**



A REAL PROPERTY AND A REAL



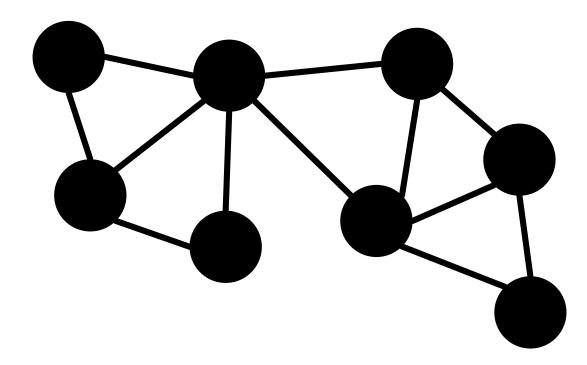
• • •





#### **Use Case 3: Clustering**





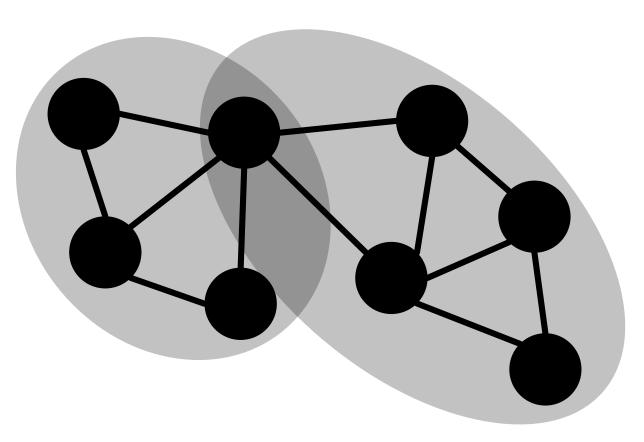
Station and the second





#### **Use Case 3: Clustering**





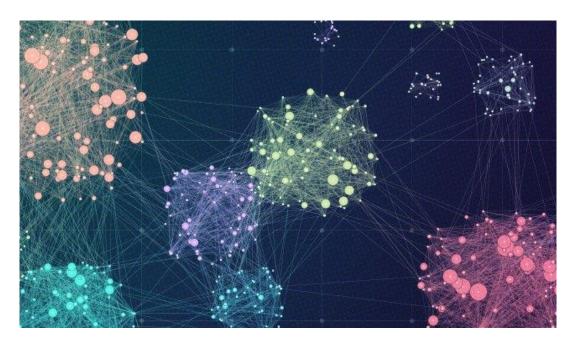




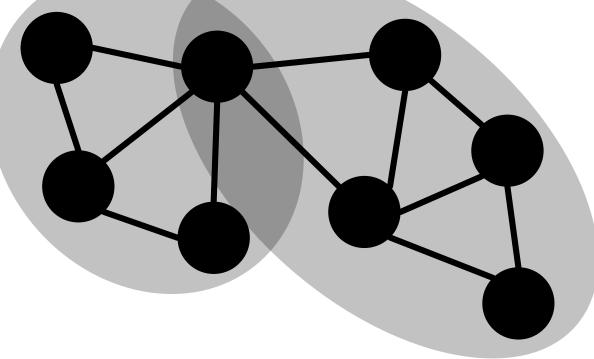
#### **Use Case 3: Clustering**

# # Clusters? Structure of clusters?

# Minibatch selection in Graph Neural Networks

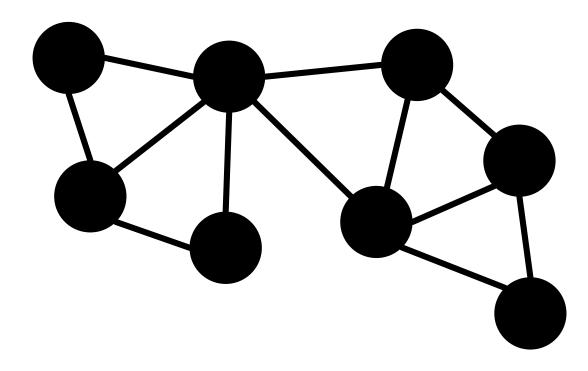








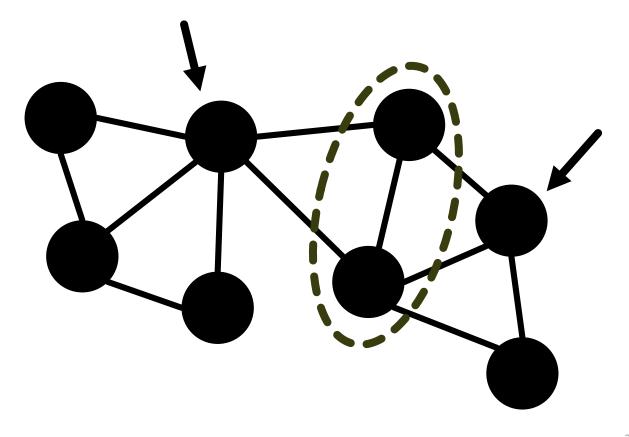




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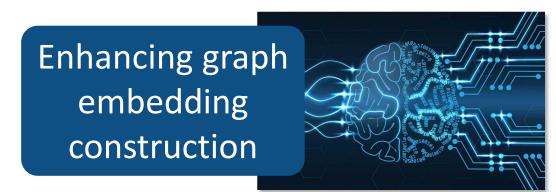


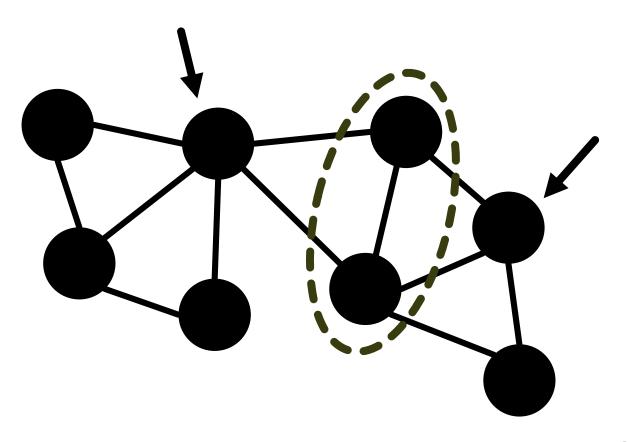


a start have







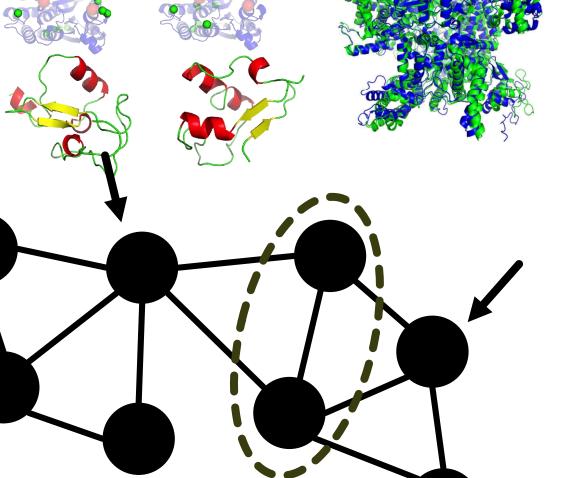


Store Level



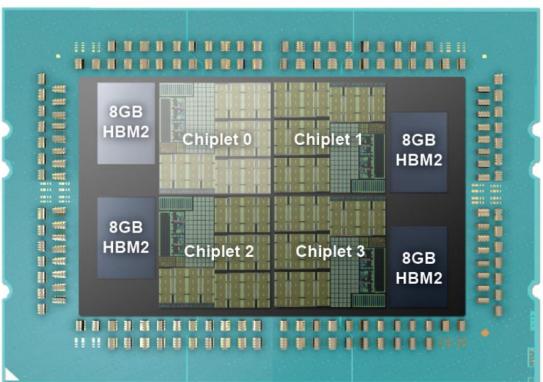


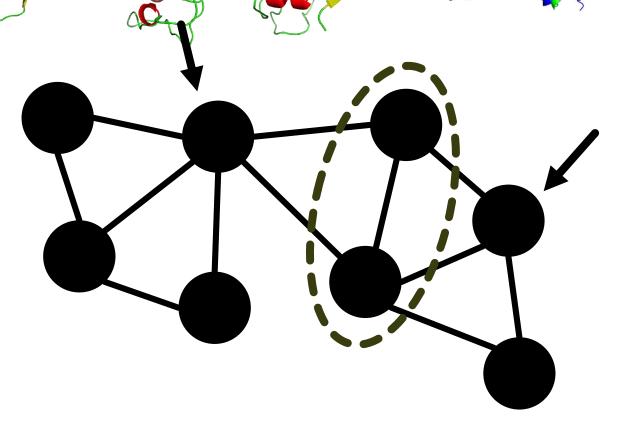
Enhancing graph embedding construction



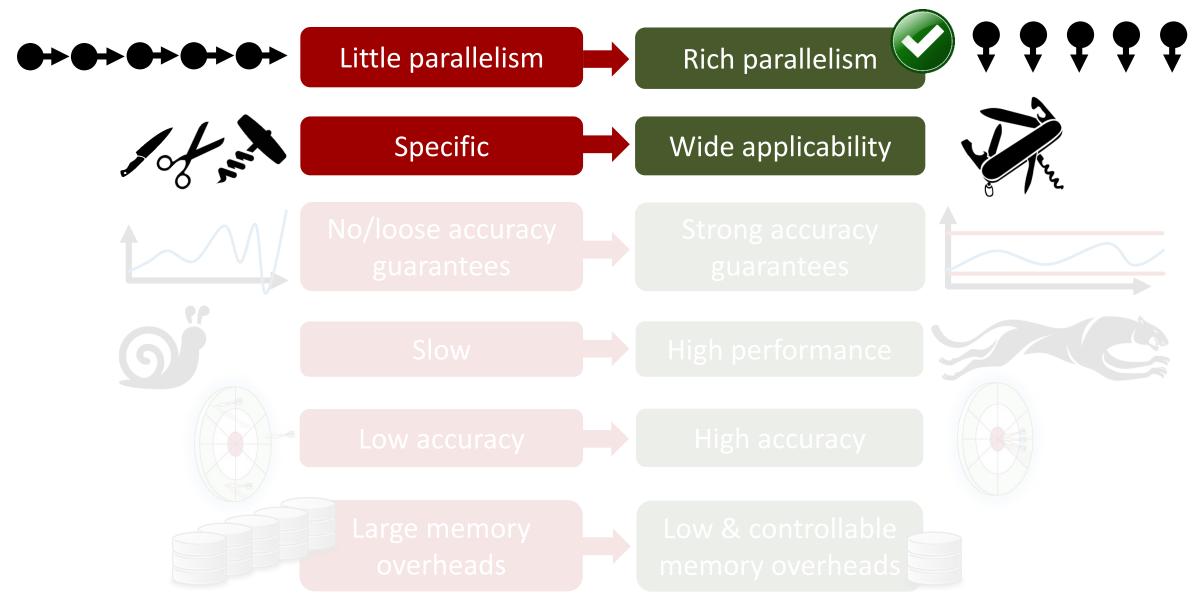


Enhancing graph embedding construction



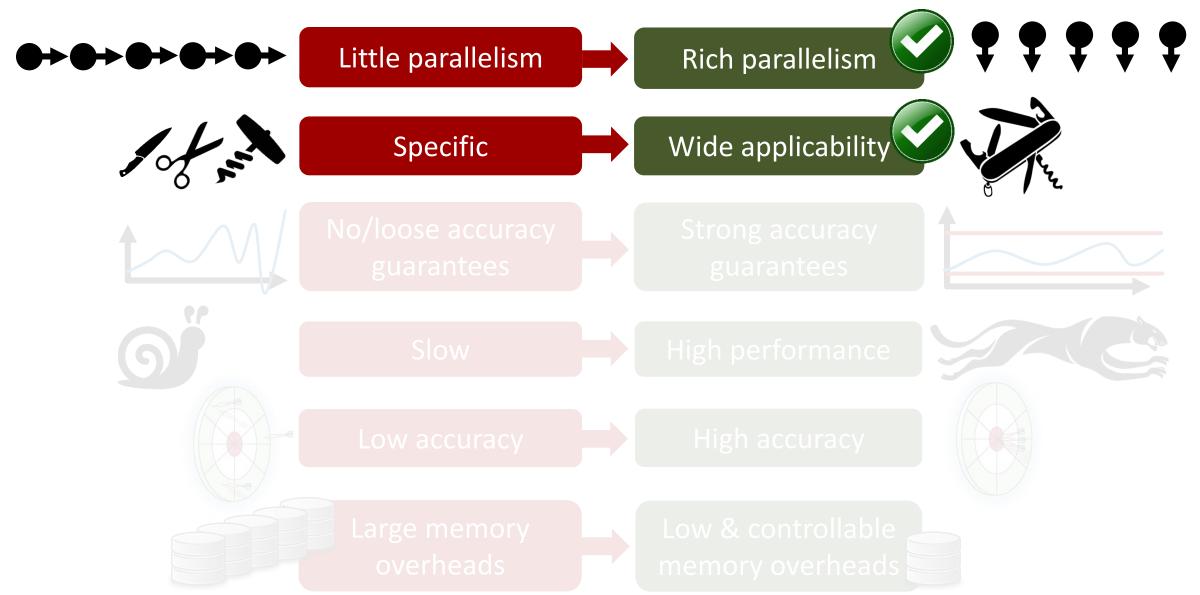




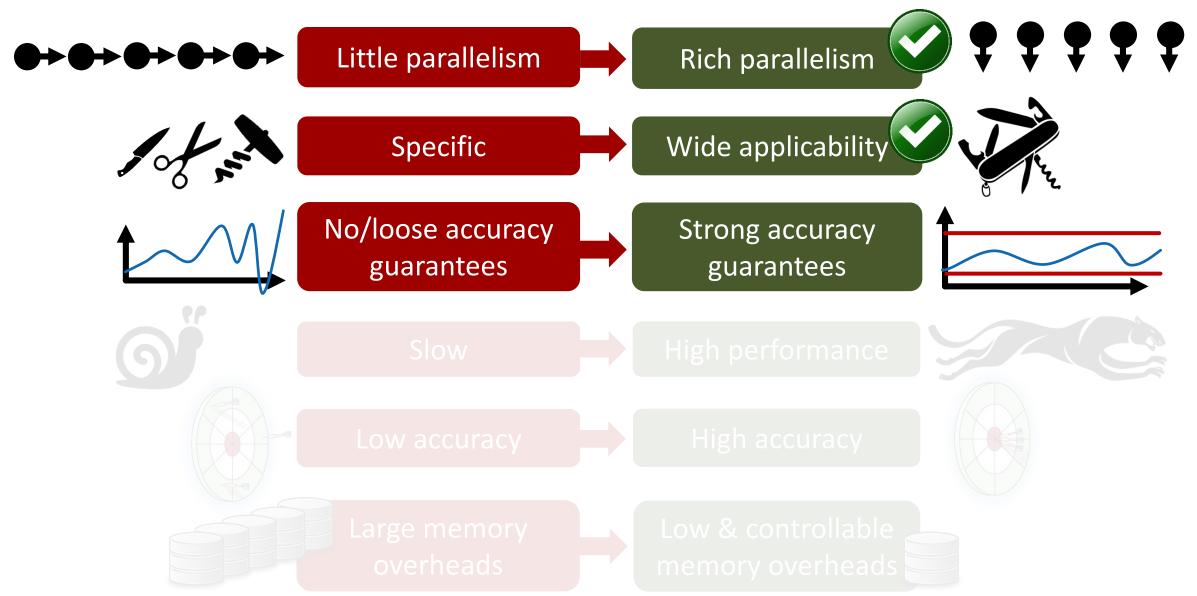


Constant and the second









a share in the second



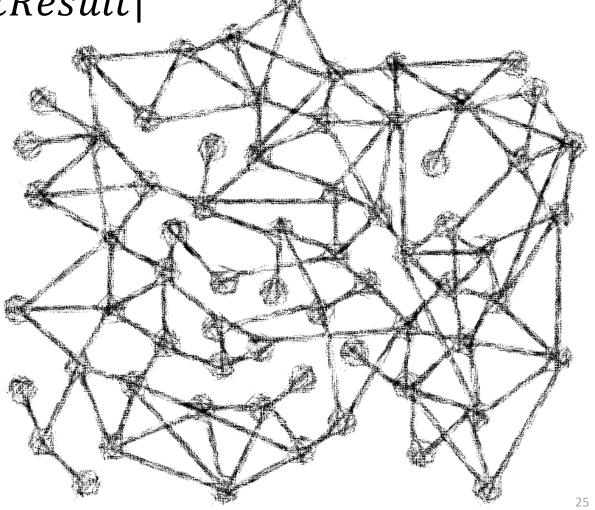
Marker & States

#### **ProbGraph: Summary of Theoretical Results**



# **ProbGraph: Summary of Theoretical Results**

We want guarantees for |*ProbGraphEstimate - exactResult*|

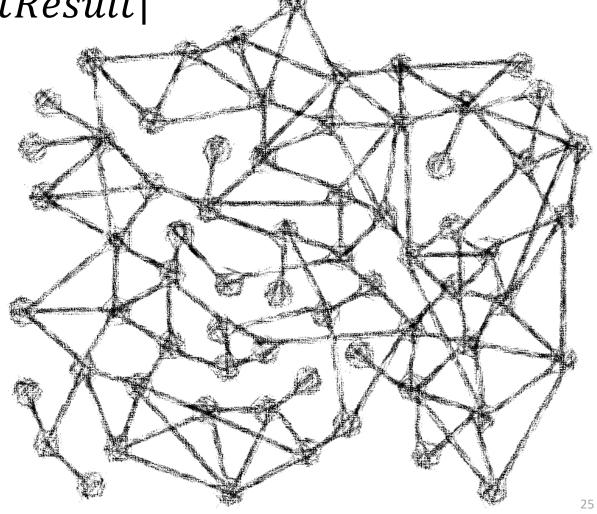




# **ProbGraph: Summary of Theoretical Results**

We want guarantees for |*ProbGraphEstimate – exactResult*|

We incorporate statistical theory of estimators





State of the state of the

## ProbGraph is asymptotically unbiased





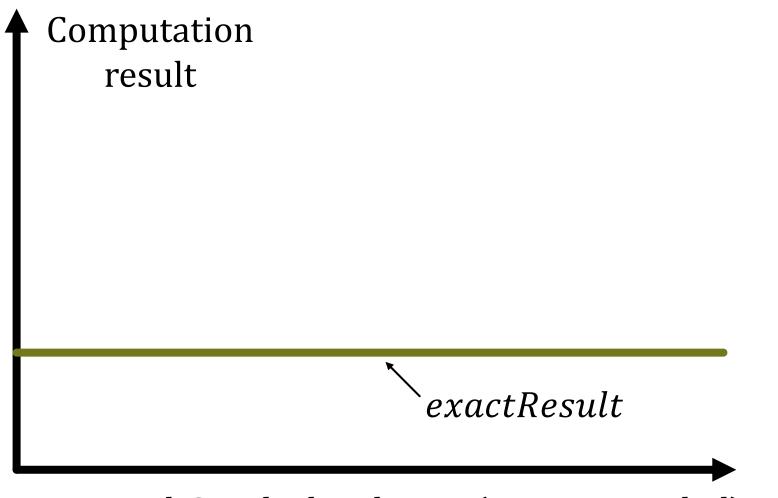
Computation result

ProbGraph sketch size (storage needed)

The second second second



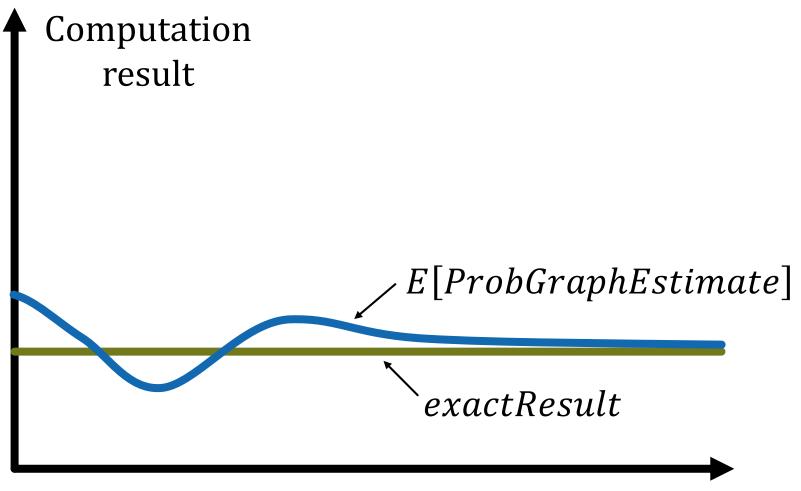




The sectors

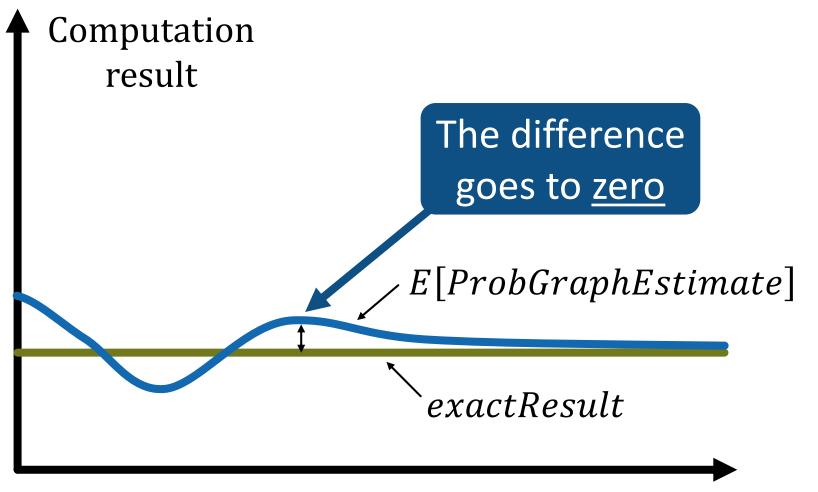
ProbGraph sketch size (storage needed)





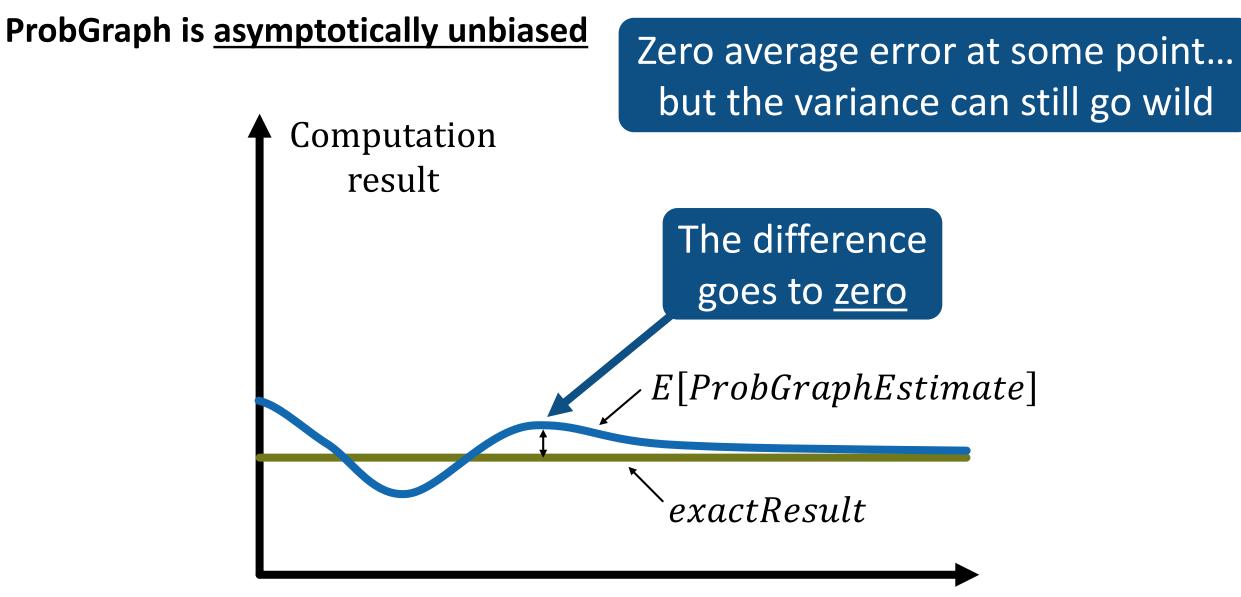
ProbGraph sketch size (storage needed)





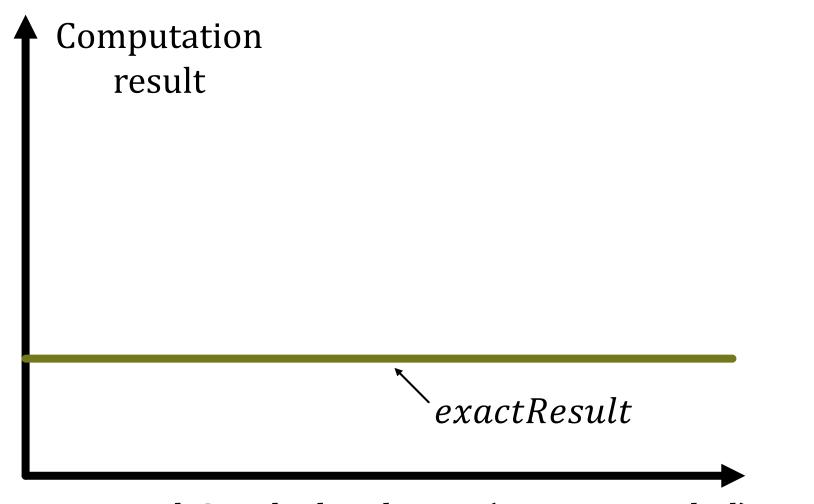
ProbGraph sketch size (storage needed)





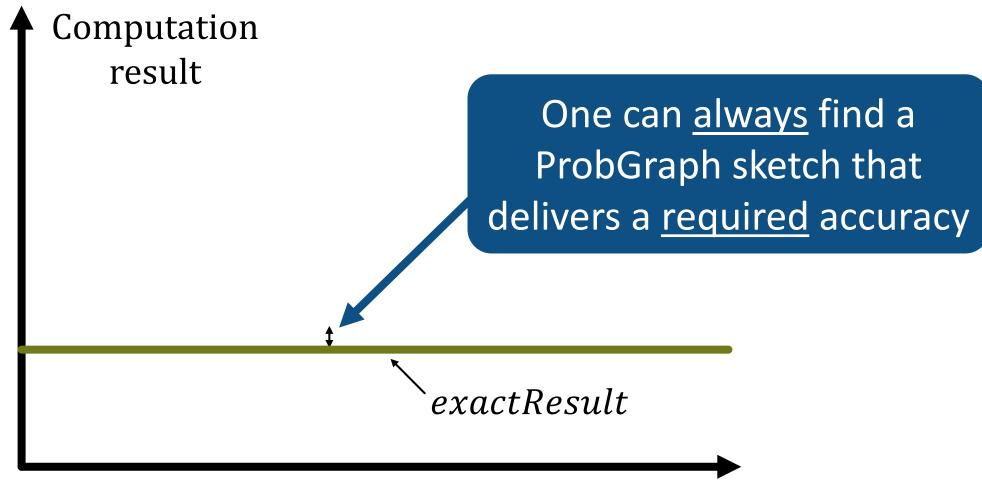




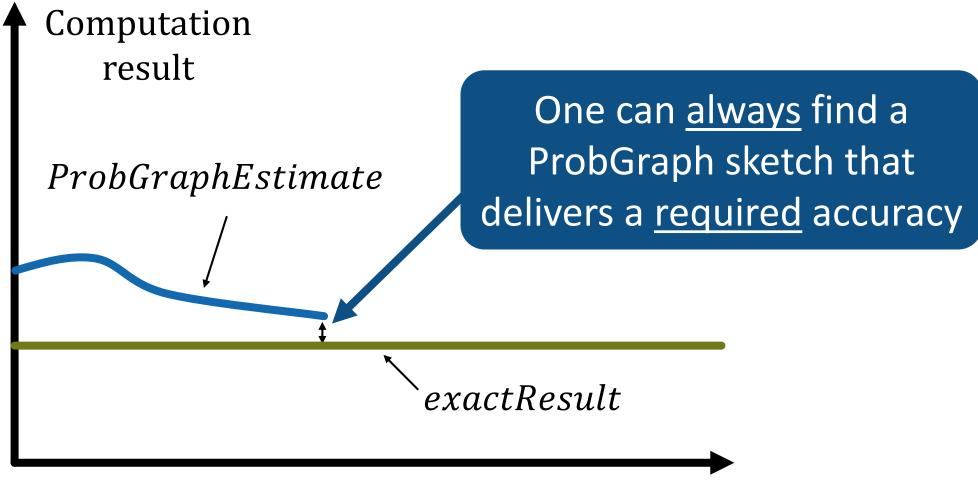


Contraction and



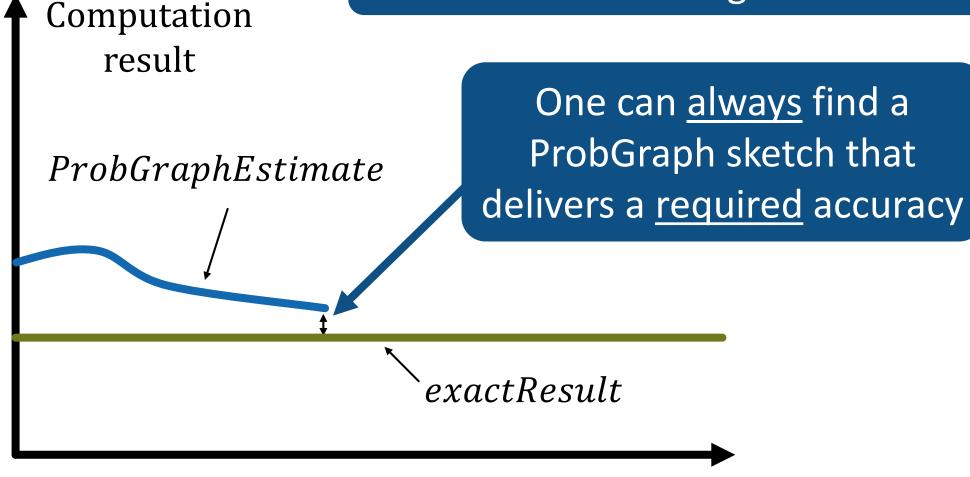






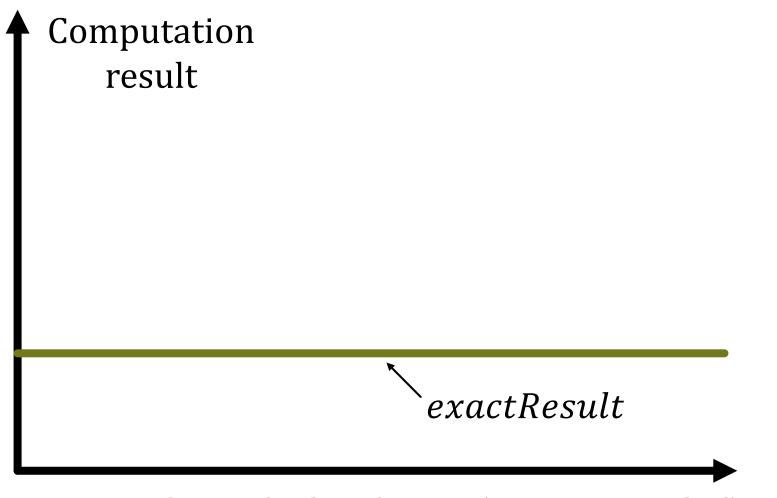


The variance also converges to zero with the increasing sketch size









The sectors



# Computation result

[1] J. Tetek, "Approximate triangle counting via sampling and fast matrix ` multiplication", arXiv 2021.

[2] S. Assadi et al., "A simple sublinear-time algorithm for counting arbitrary subgraphs via edge sampling", arXiv 2018.

[3] T. Eden et al., "Approximately counting triangles in sublinear time", SIAM Journal on Computing, 2017.

[4] B. Bandyopadhyay et al., "Topological graph sketching for incremental and scalable analytics", CIKM, 2016.

[5] R. Pagh et al., "Colorful triangle counting and a MapReduce implementation", Information Processing Letters, 2012.

[6] O. Papapetrou et al., "Cardinality estimation and dynamic length adaptation for bloom filters", Distributed and Parallel Databases, 2010.

[7] C. E. Tsourakakis et al., "Doulion: counting triangles in massive graphs with a coin". ACM KDD. 2009.

[8] M. Besta et al., "Slim graph: Practical lossy graph compression for approximate graph processing, storage, and analytics". ACM/IEEE SC. 2019

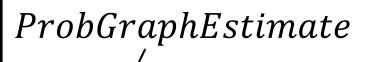
`exactResult

ProbGraph sketch size (storage needed)

[1-8]

Other estimators

# Computation result



Other estimators

[1] J. Tetek, "Approximate triangle counting via sampling and fast matrix ` multiplication", arXiv 2021.

[2] S. Assadi et al., "A simple sublinear-time algorithm for counting arbitrary subgraphs via edge sampling", arXiv 2018.

[3] T. Eden et al., "Approximately counting triangles in sublinear time", SIAM Journal on Computing, 2017.

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`exactResult

ProbGraph sketch size (storage needed)

[1-8]

# Computation result

# ProbGraphEstimate

Other estimators

[1] J. Tetek, "Approximate triangle counting via sampling and fast matrix ` multiplication", arXiv 2021.

[2] S. Assadi et al., "A simple sublinear-time algorithm for counting arbitrary subgraphs via edge sampling", arXiv 2018.

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[4] B. Bandyopadhyay et al., "Topological graph sketching for incremental and scalable analytics", CIKM, 2016.

[5] R. Pagh et al., "Colorful triangle counting and a MapReduce implementation", Information Processing Letters, 2012.

[6] O. Papapetrou et al., "Cardinality estimation and dynamic length adaptation for bloom filters", Distributed and Parallel Databases, 2010.

[7] C. E. Tsourakakis et al., "Doulion: counting triangles in massive graphs with a coin". ACM KDD. 2009.

[8] M. Besta et al., "Slim graph: Practical lossy graph compression for approximate graph processing, storage, and analytics". ACM/IEEE SC. 2019

<u>No other consistent estimator</u> has lower MSE / variance `exactResult

ProbGraph sketch size (storage needed)

[1-8]



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## **ProbGraph has strong concentration bounds**

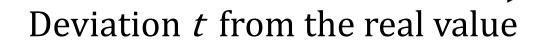




at the second second



P(|ProbGraphEstimate





P(|ProbGraphEstimate

This probability decreases <u>exponentially</u> fast

Deviation *t* from the real value



P(|ProbGraphEstimate

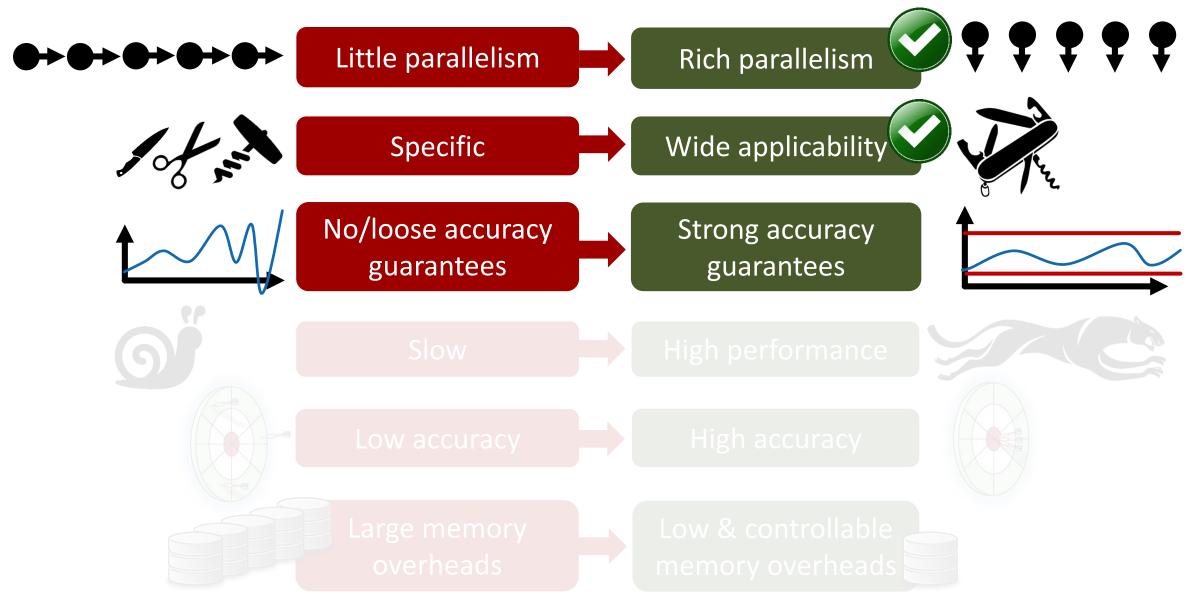
ProbGraph is unlikely to deviate much from the true values

This probability decreases <u>exponentially</u> fast

Deviation *t* from the real value



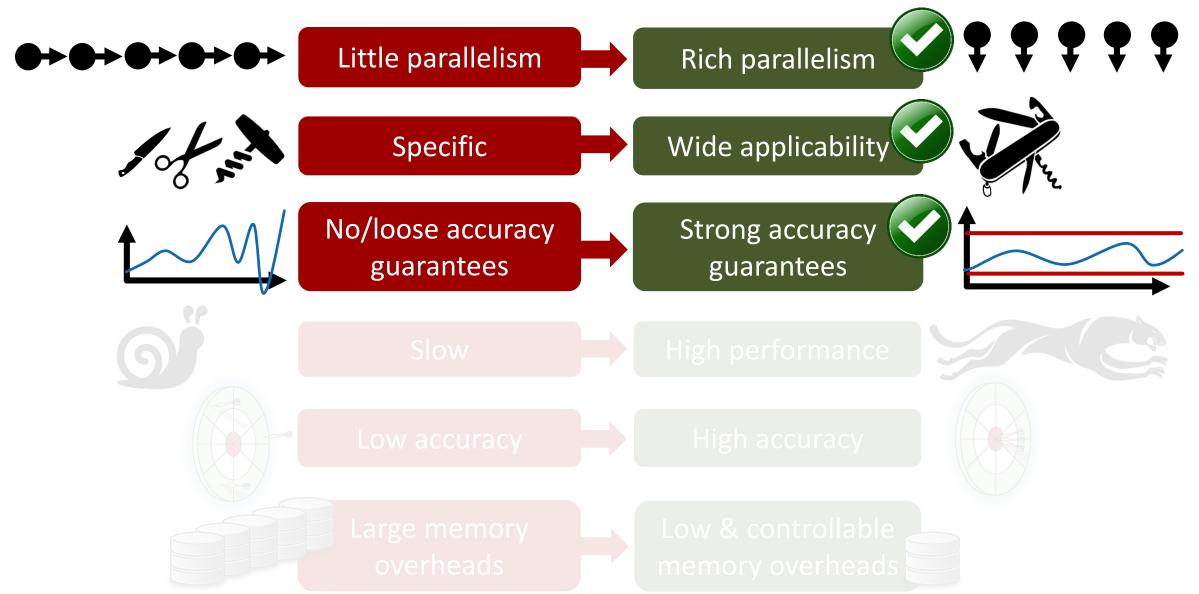
# **Approximate Graph Processing: Our Objectives**



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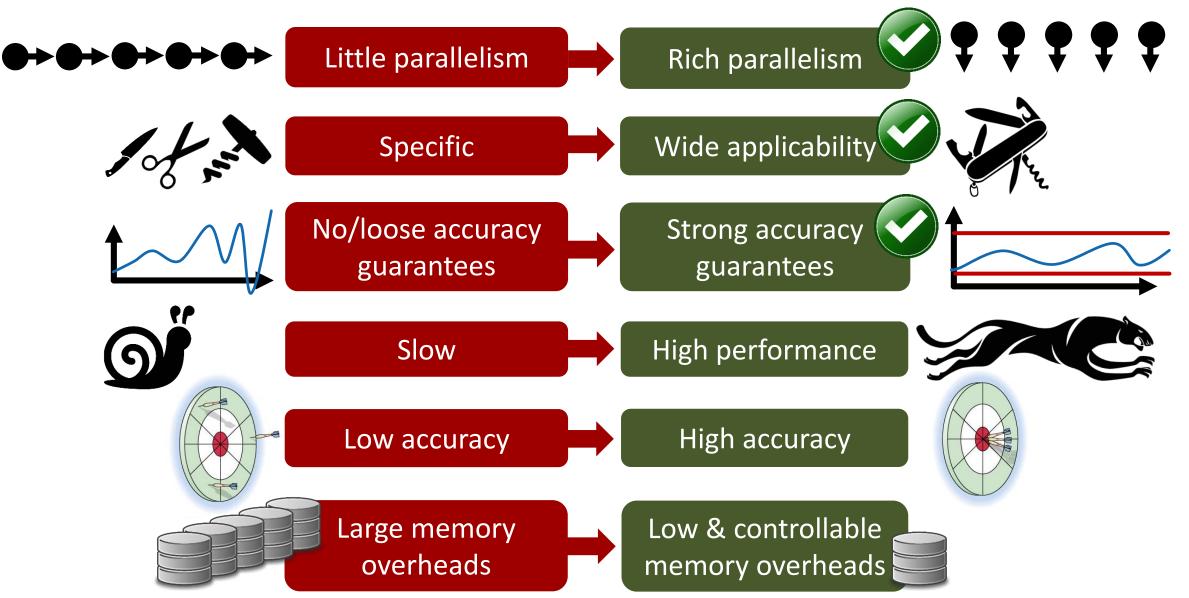
## **Approximate Graph Processing: Our Objectives**



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## **Approximate Graph Processing: Our Objectives**



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## **Evaluation: Used Machines & Objectives**

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## **Evaluation: Used Machines & Objectives**

CSCS Cray Piz Daint, 64 GB per compute node

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## **Evaluation: Used Machines & Objectives**

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Dell PowerEdge R910 server

CSCS Cray Piz Daint, 64 GB per compute node

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#### **Evaluation: Used Machines & Objectives**

# **Goal**: One design with... <u>large</u> speedups + <u>small & controlled</u> accuracy loss + <u>small & controlled</u> memory requirements

Ø

Dell PowerEdge R910 server

CSCS Cray Piz Daint, 64 GB per compute node





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## **Considered Graph Datasets**





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#### **Considered Graph Datasets**

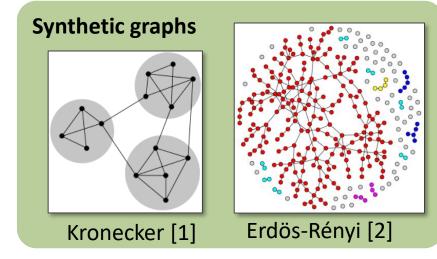
67 graph datasets, 15 areas, 5 major graph dataset repositories

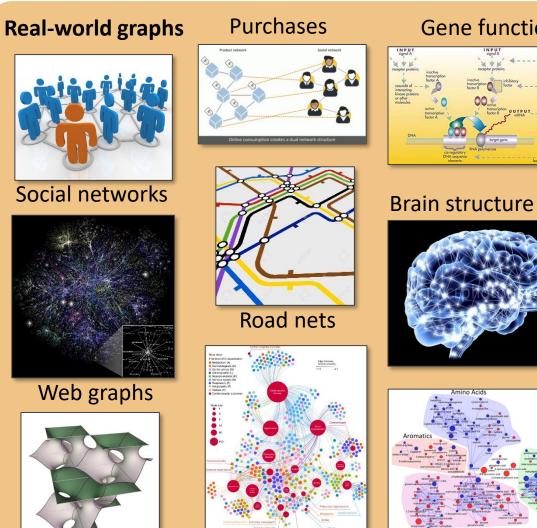


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## **Considered Graph Datasets**

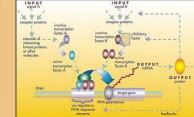
67 graph datasets, 15 areas, 5 major graph dataset repositories





Medicine

Gene functions



Chemistry



Communication



Citation graphs



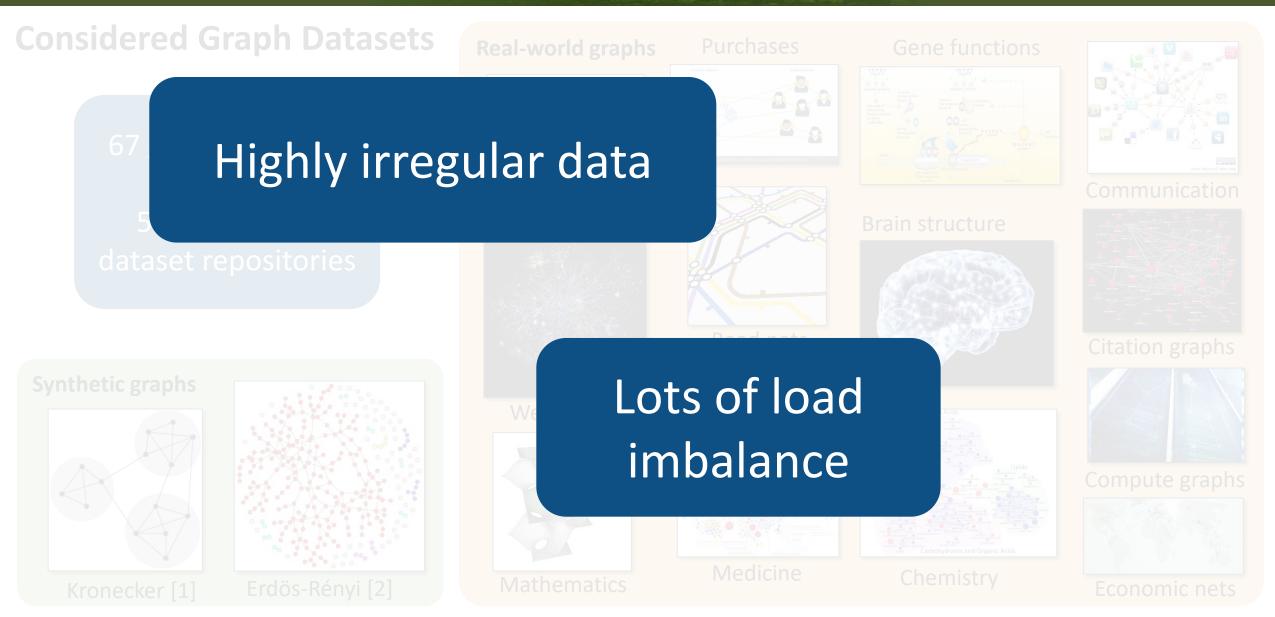
Compute graphs



[1] J. Leskovec et al. Kronecker Graphs: An Approach to Modeling Networks. J. Mach. Learn. Research. 2010. [2] P. Erdos and A. Renyi. On the evolution of random graphs. Pub. Math. Inst. Hun. A. Science. 1960.

**Mathematics** 





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[1] J. Leskovec et al. Kronecker Graphs: An Approach to Modeling Networks. J. Mach. Learn. Research. 2010.[2] P. Erdos and A. Renyi. On the evolution of random graphs. Pub. Math. Inst. Hun. A. Science. 1960.





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## **Triangle Counting**

Cores/threads: 32 Max memory overhead: 20%

# **Triangle Counting**

Cores/threads: 32 Max memory overhead: 20%

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# **Triangle Counting**

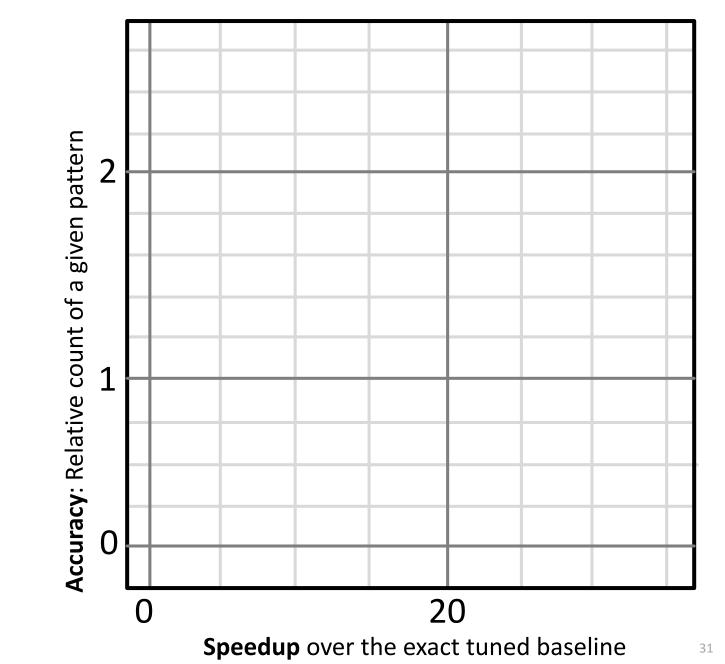
Cores/threads: 32 Max memory overhead: 20%

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| Speedup over the exact tuned baseline |  |  |  |  |  |  |   |  |  |

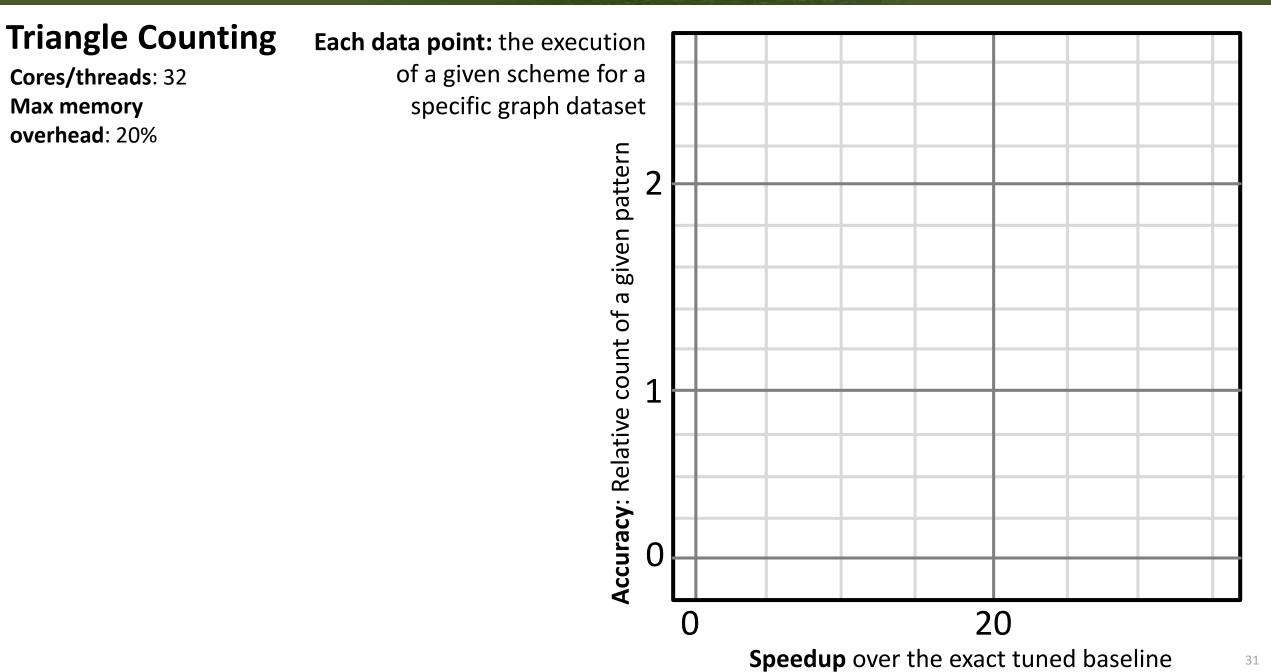
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# **Triangle Counting**

Cores/threads: 32 Max memory overhead: 20%

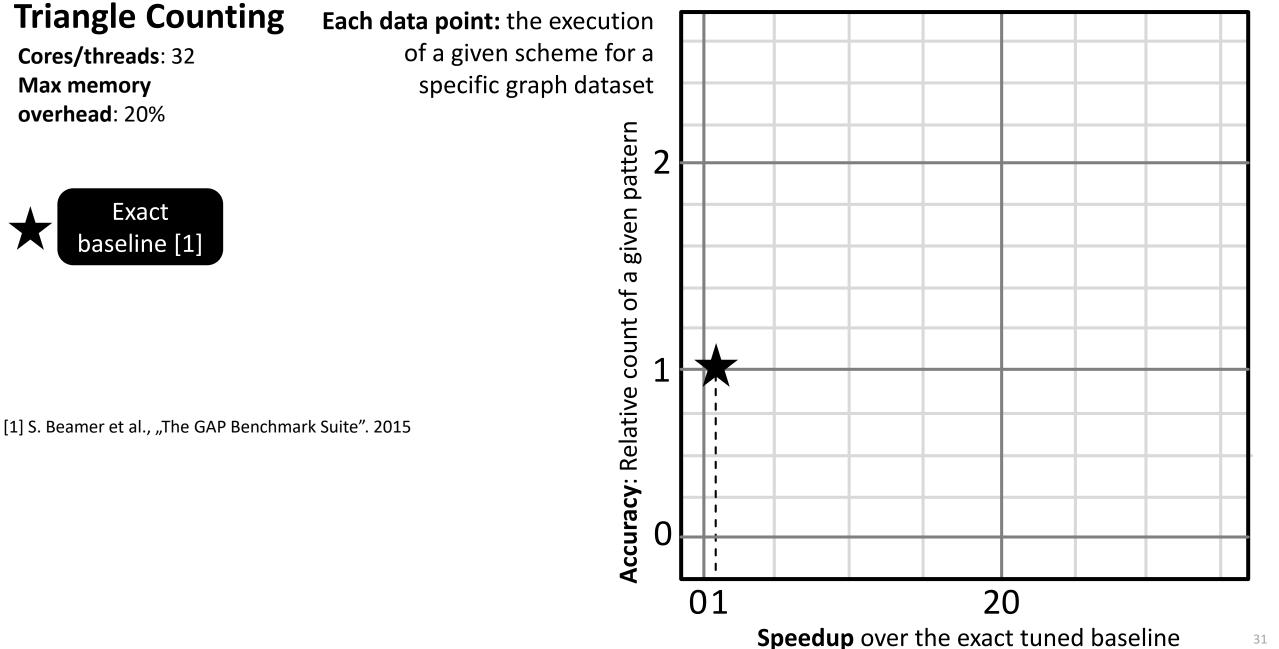


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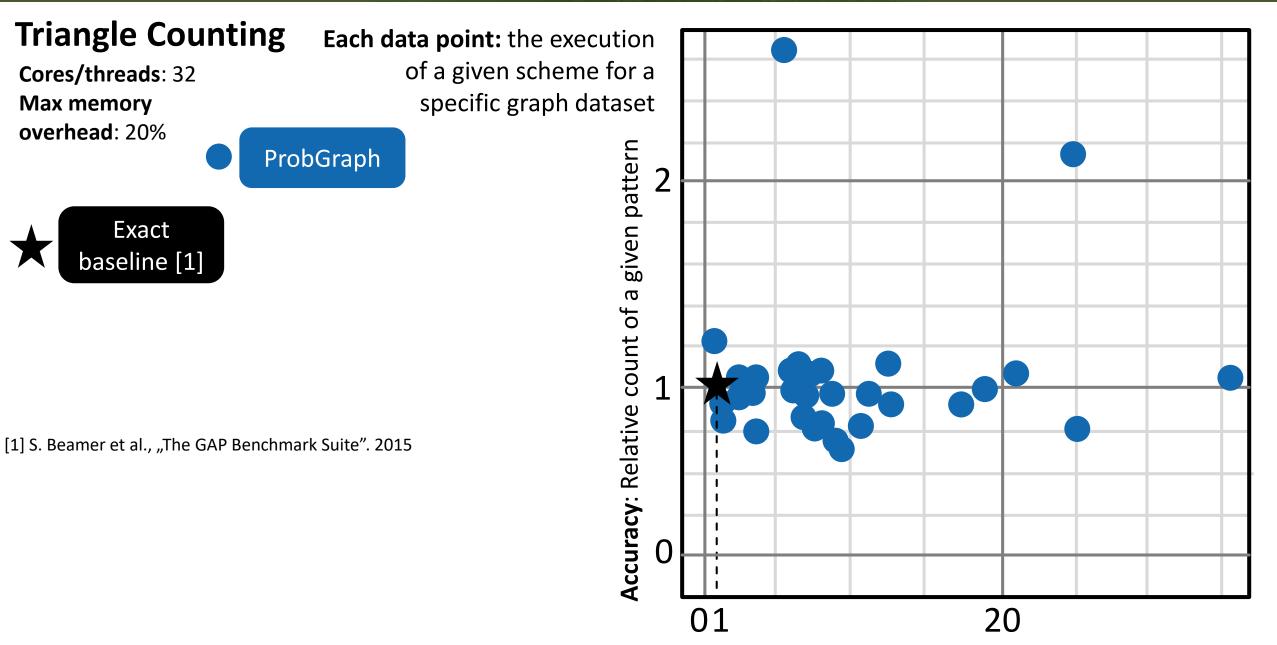
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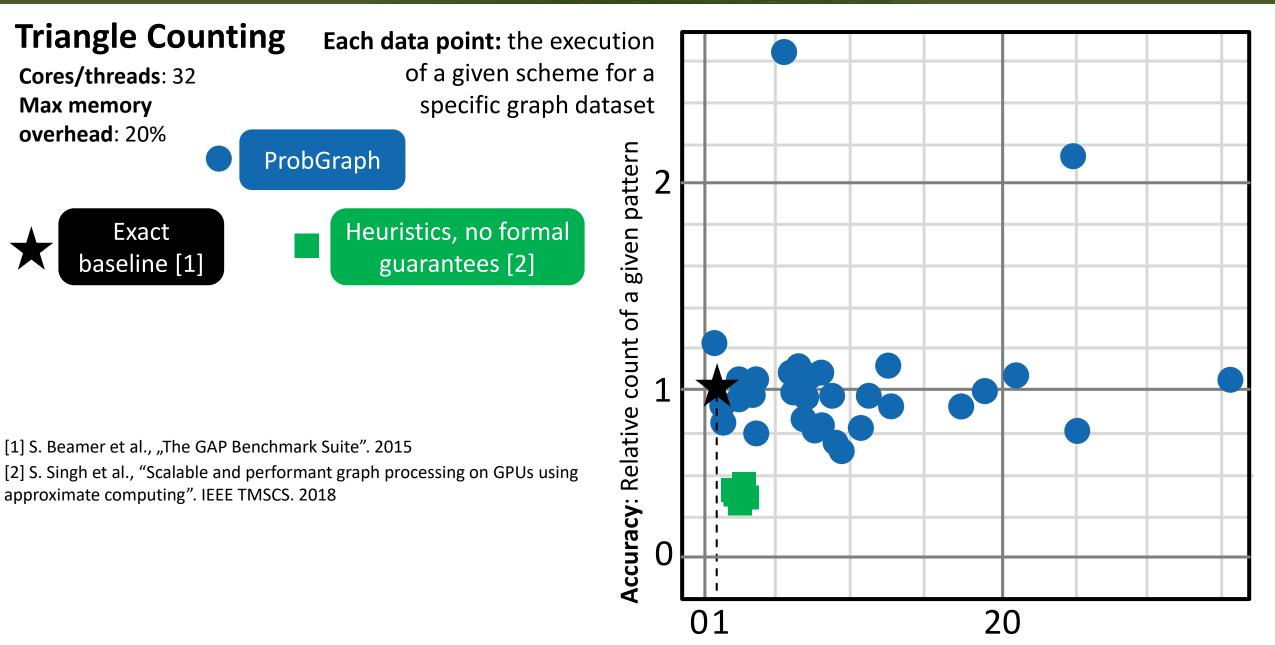
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Speedup over the exact tuned baseline 31

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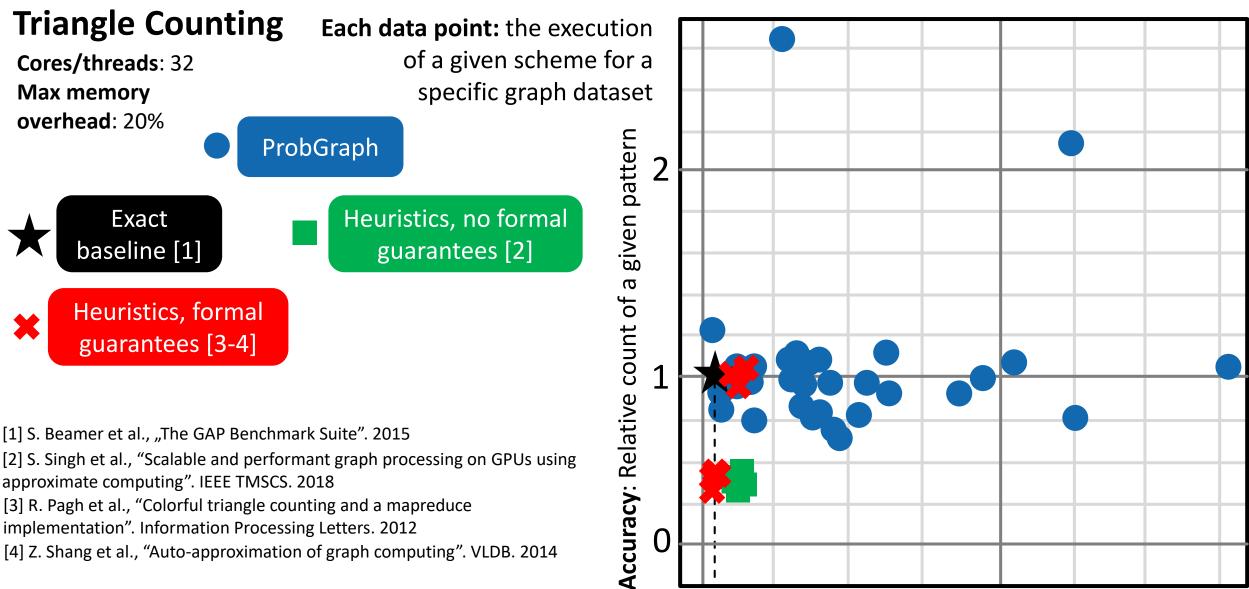


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Speedup over the exact tuned baseline

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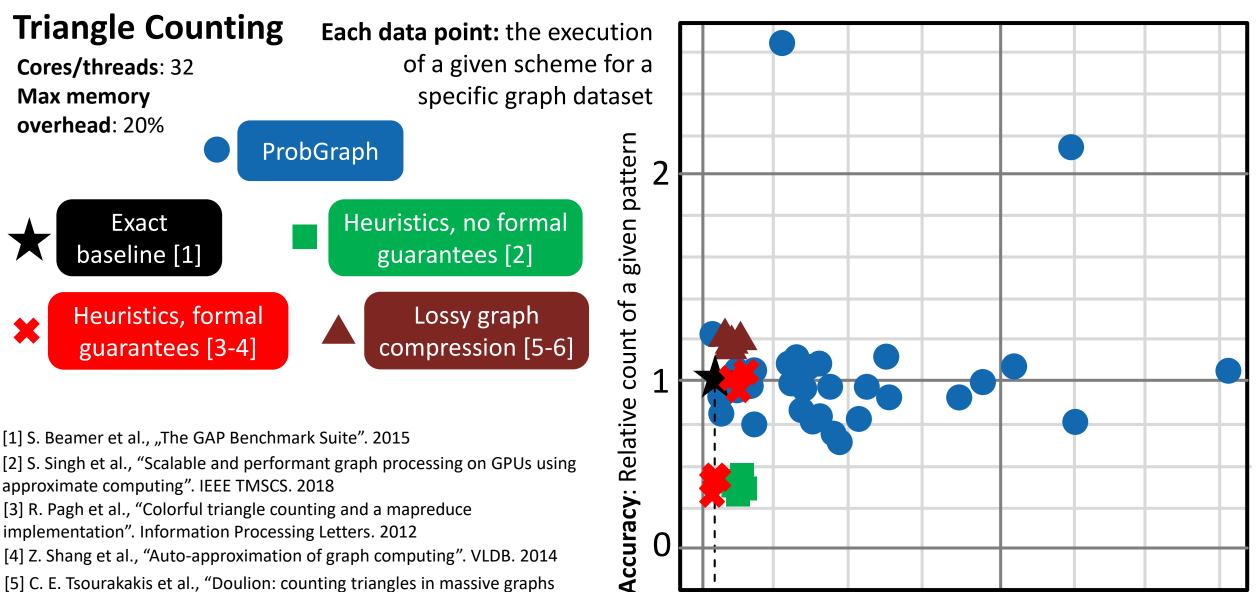
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[4] Z. Shang et al., "Auto-approximation of graph computing". VLDB. 2014

**Speedup** over the exact tuned baseline

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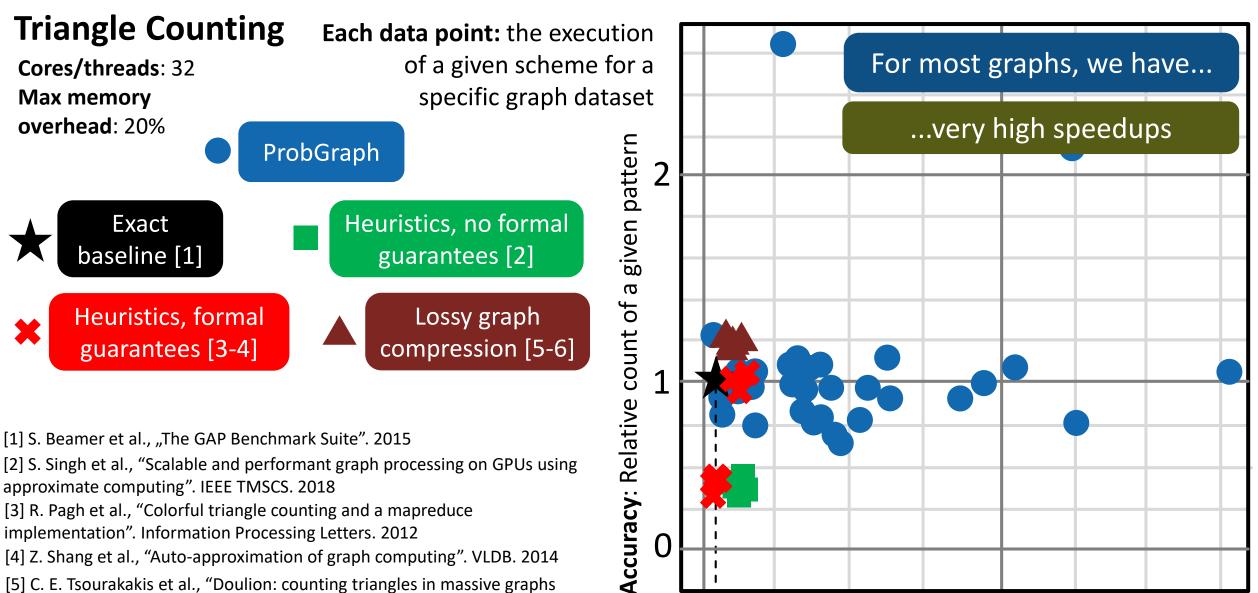
- [3] R. Pagh et al., "Colorful triangle counting and a mapreduce implementation". Information Processing Letters. 2012
- [4] Z. Shang et al., "Auto-approximation of graph computing". VLDB. 2014
- [5] C. E. Tsourakakis et al., "Doulion: counting triangles in massive graphs with a coin". ACM KDD. 2009.
- [6] M. Besta et al., "Slim graph: Practical lossy graph compression for approximate graph processing, storage, and analytics". ACM/IEEE SC. 2019

**Speedup** over the exact tuned baseline

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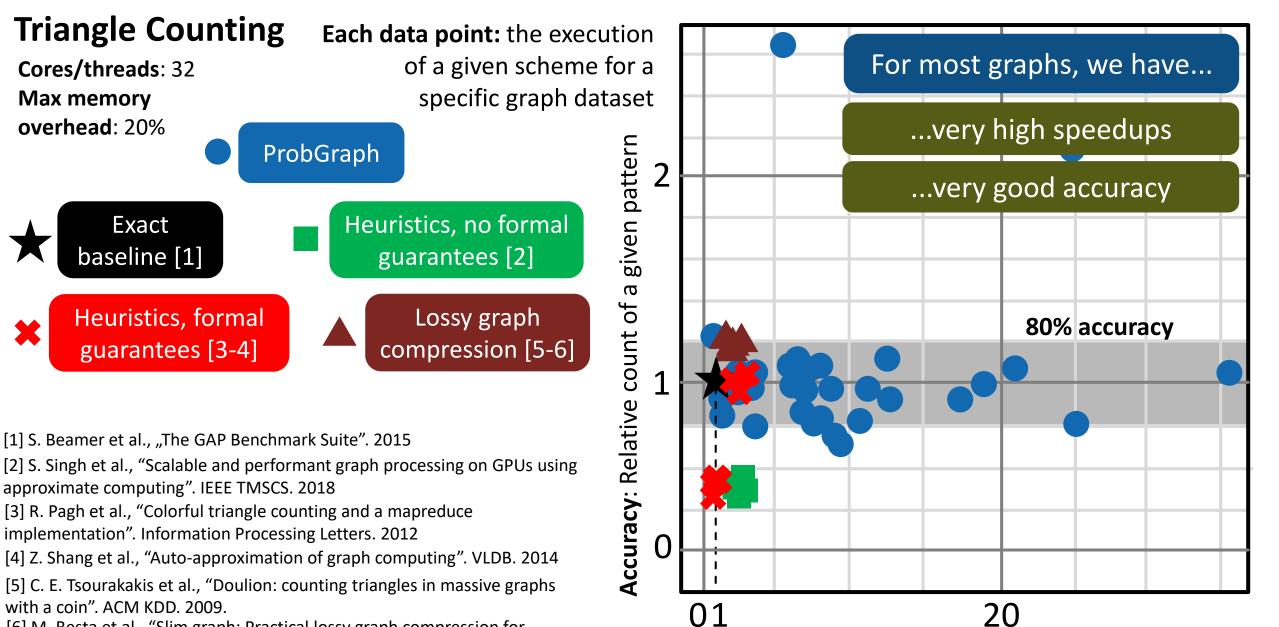
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**Speedup** over the exact tuned baseline

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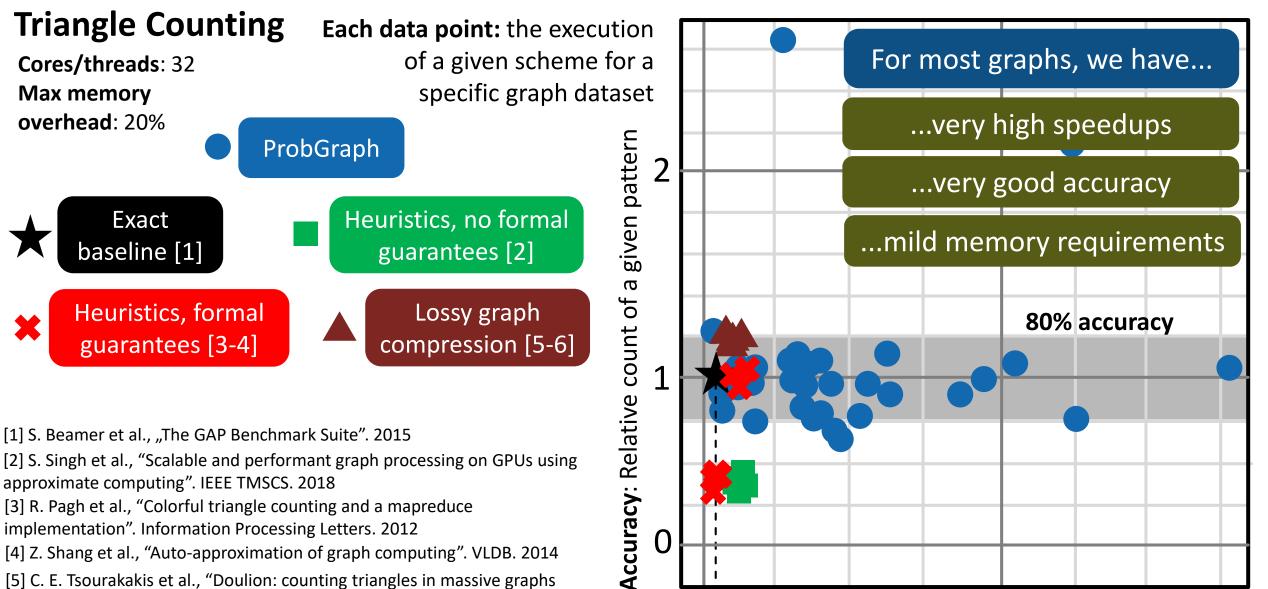


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[6] M. Besta et al., "Slim graph: Practical lossy graph compression for approximate graph processing, storage, and analytics". ACM/IEEE SC. 2019

Speedup over the exact tuned baseline

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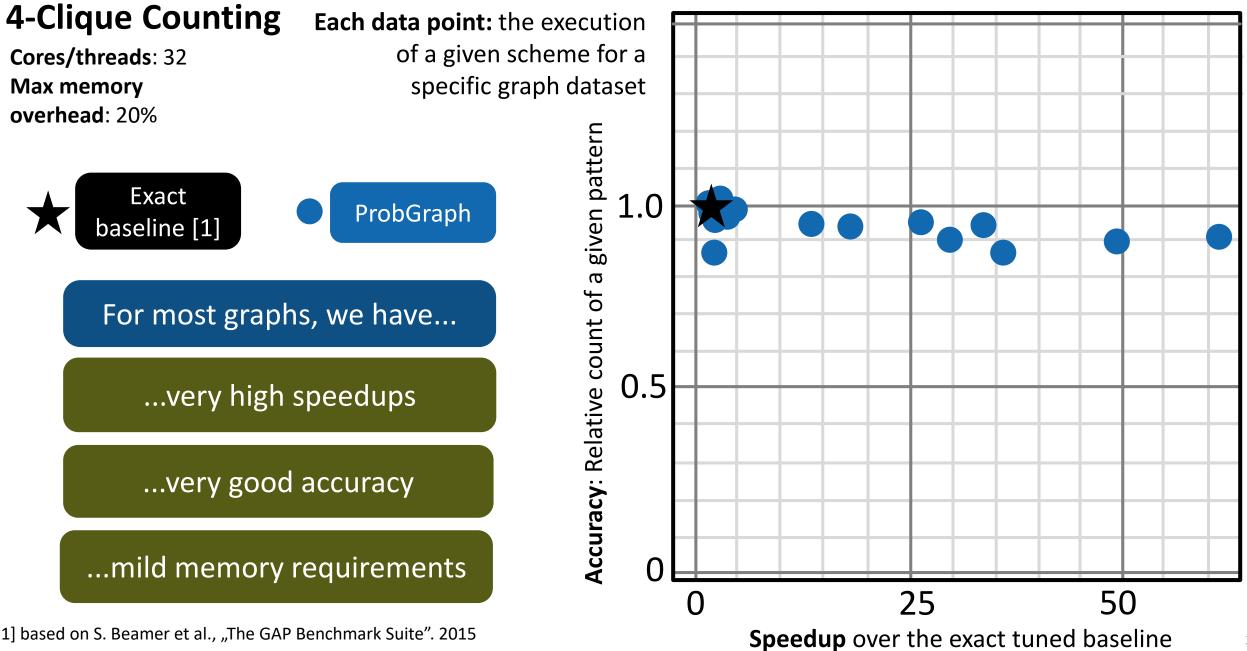
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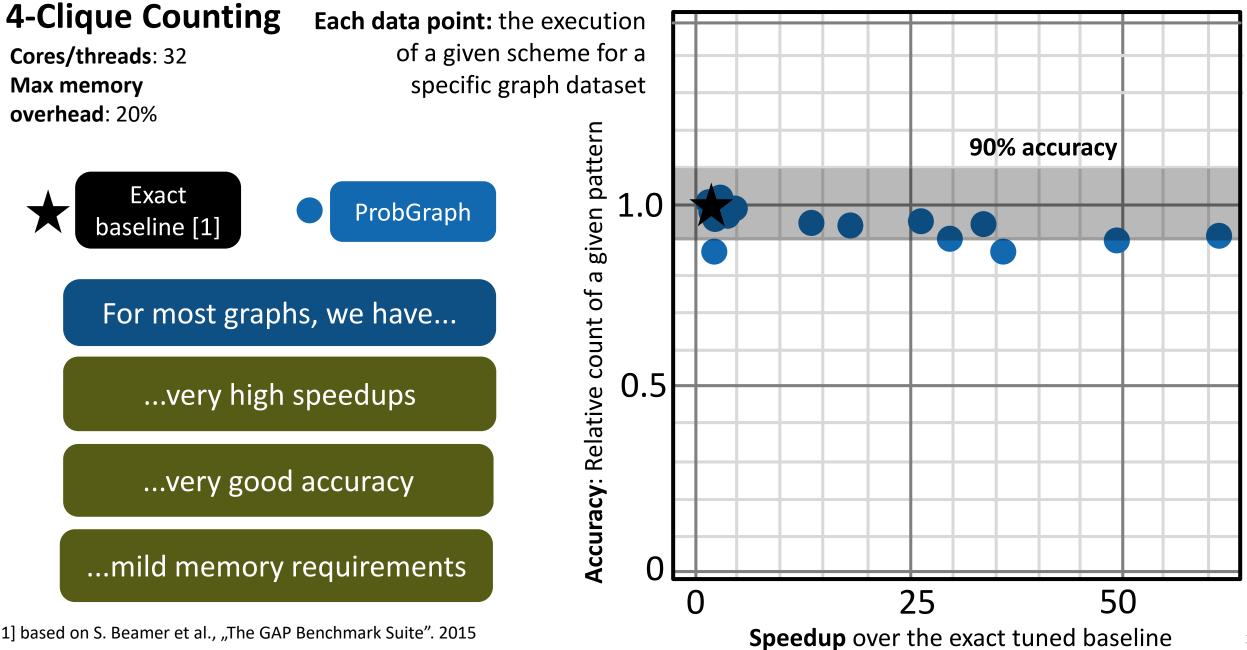


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[1] based on S. Beamer et al., "The GAP Benchmark Suite". 2015

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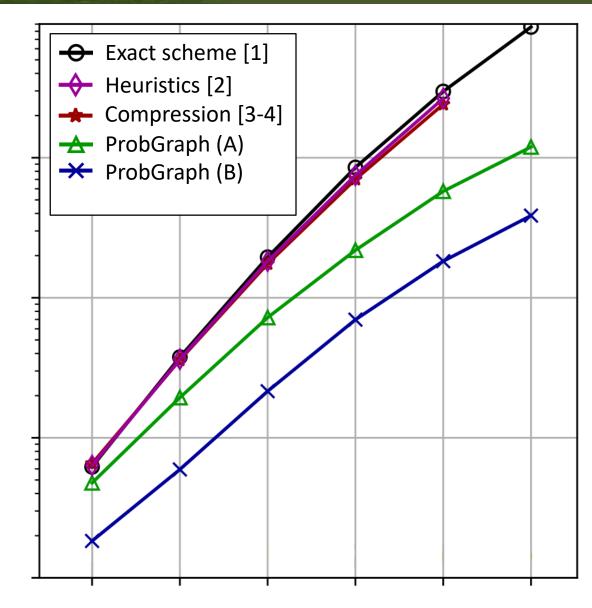


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[1] based on S. Beamer et al., "The GAP Benchmark Suite". 2015

# **Clustering (Scaling)**

Max memory overhead: 20%

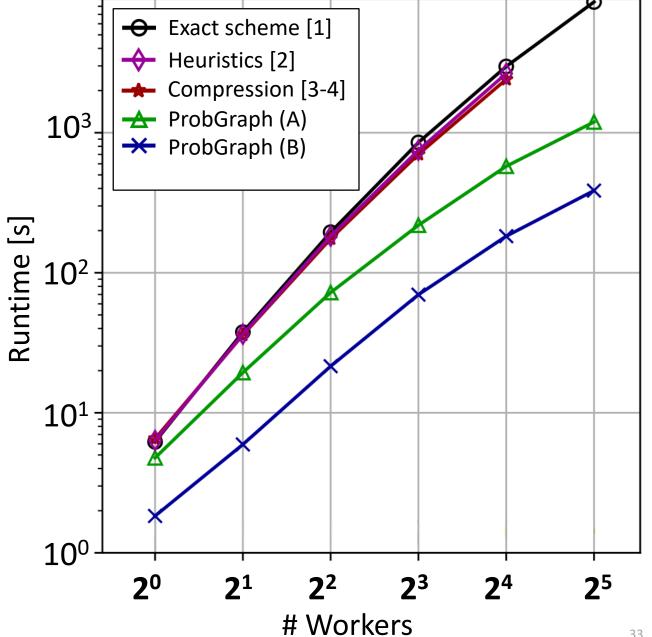


[1] S. Beamer et al., "The GAP Benchmark Suite". 2015
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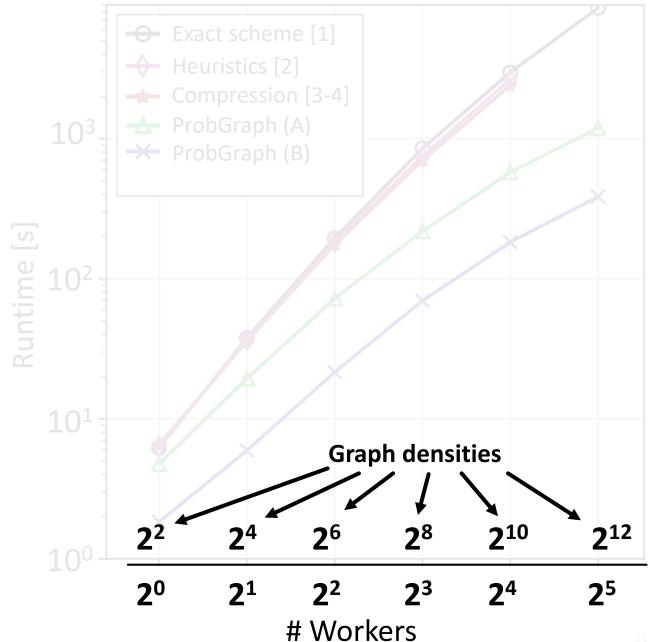
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# **Clustering (Scaling)**

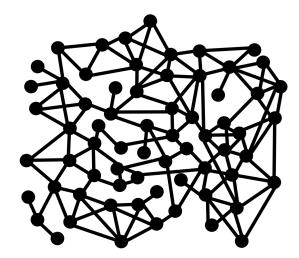


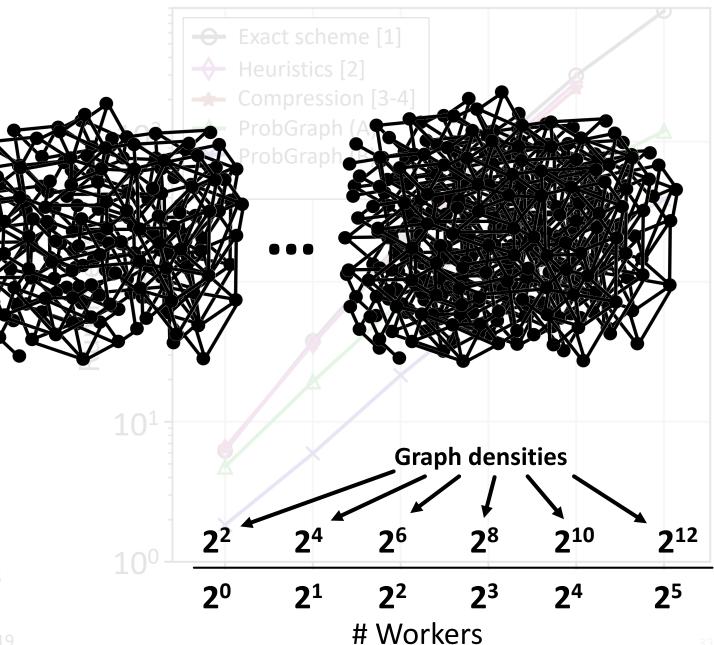
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# **Clustering (Scaling)**

Max memory overhead: 20%





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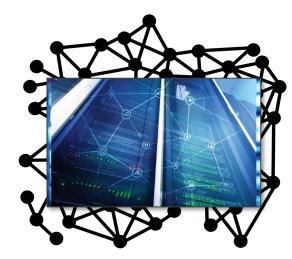
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[3] C. E. Tsourakakis et al., "Doulion: counting triangles in mass

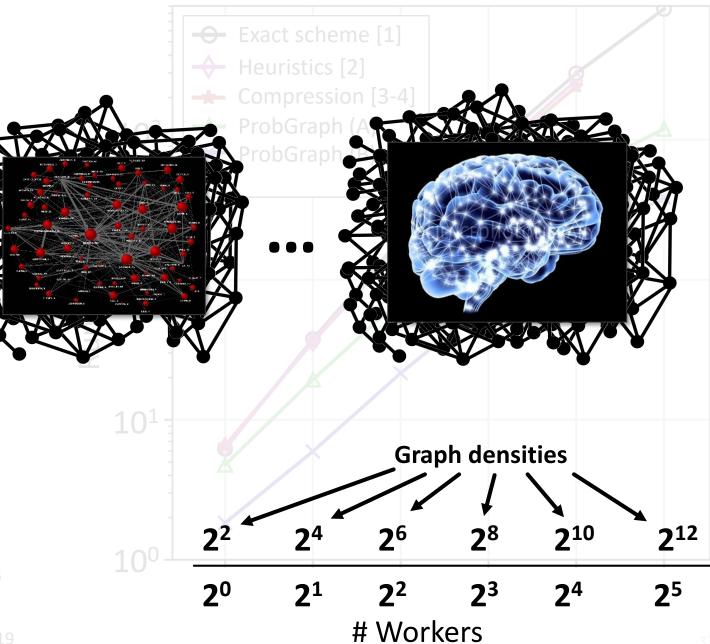
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Max memory overhead: 20%





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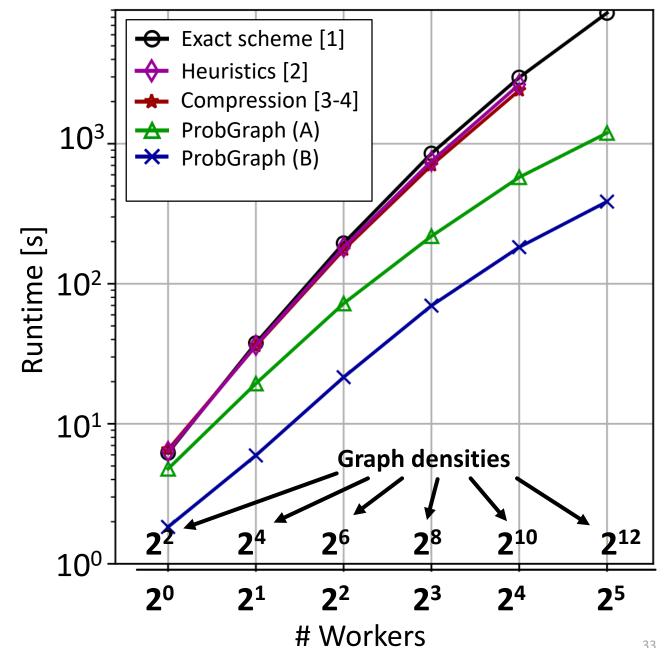
 [2] S. Beamer et al., "The GAP Benchmark Suite 12015
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 [2] C. F. Taguradadia et al., "Daulian source triangles in response to the second secon

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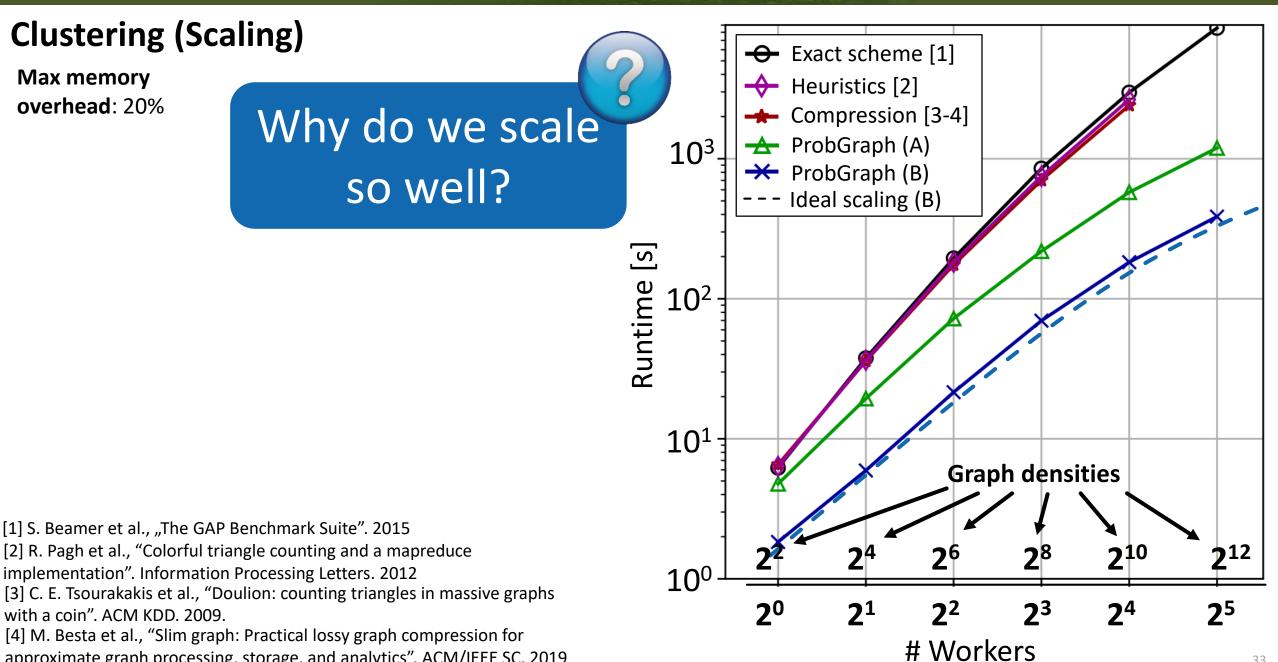
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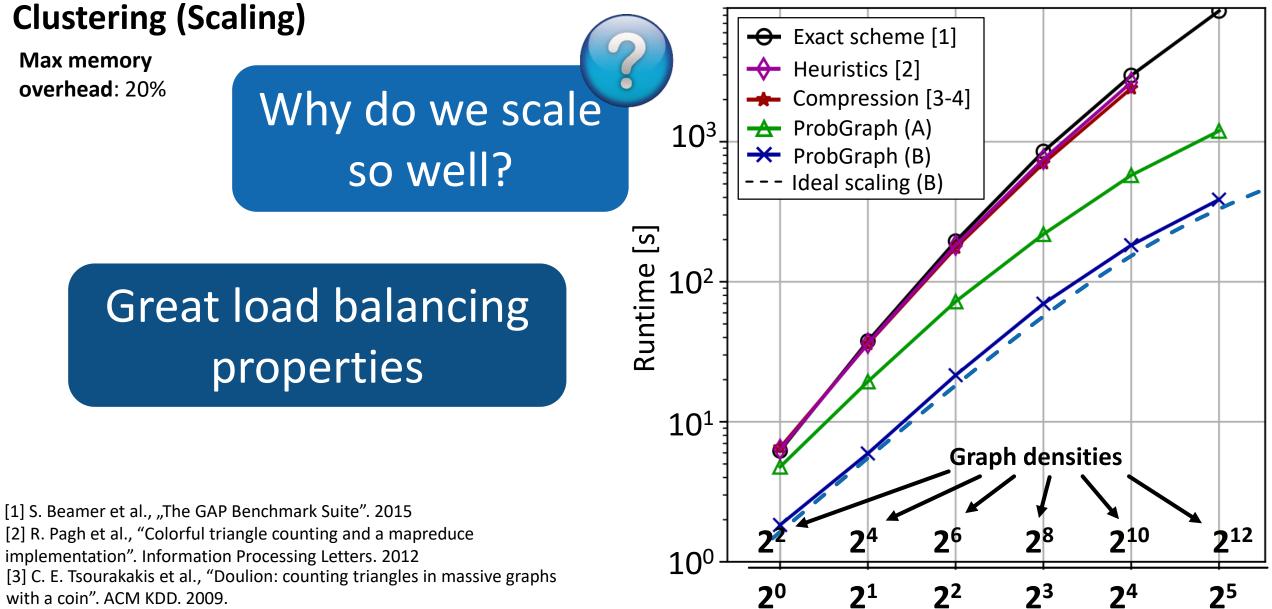


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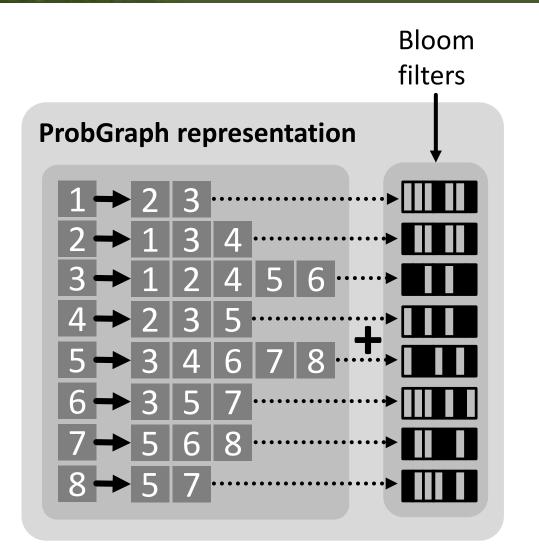
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[4] M. Besta et al., "Slim graph: Practical lossy graph compression for approximate graph processing, storage, and analytics". ACM/IEEE SC. 2019

# Workers

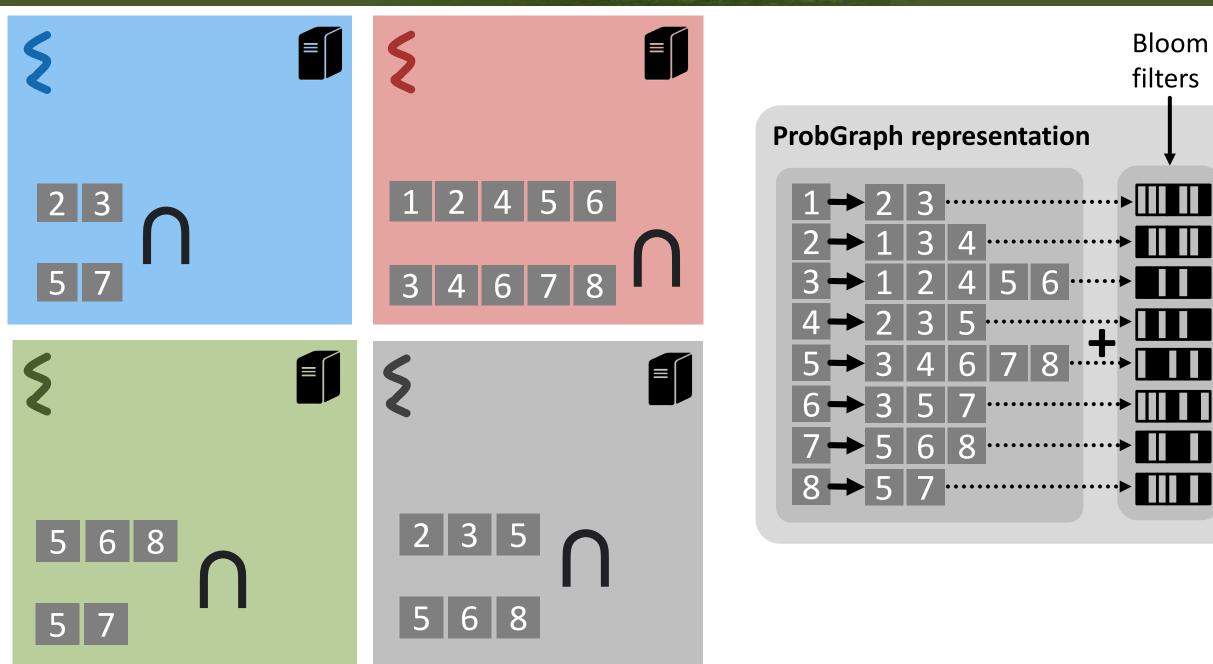




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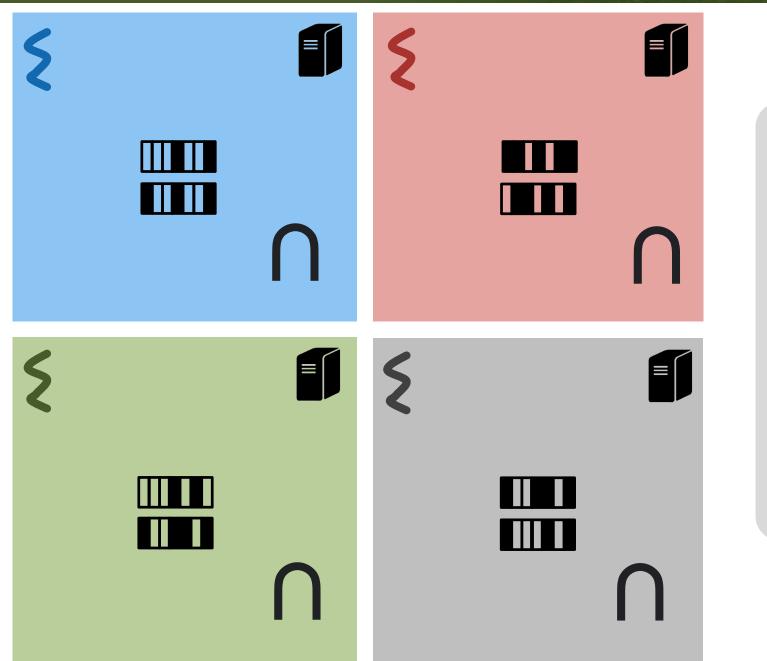


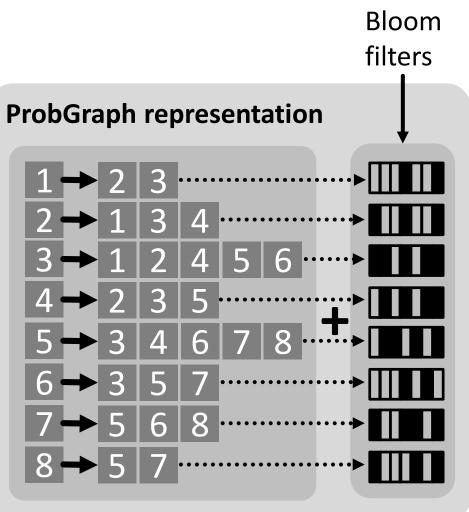
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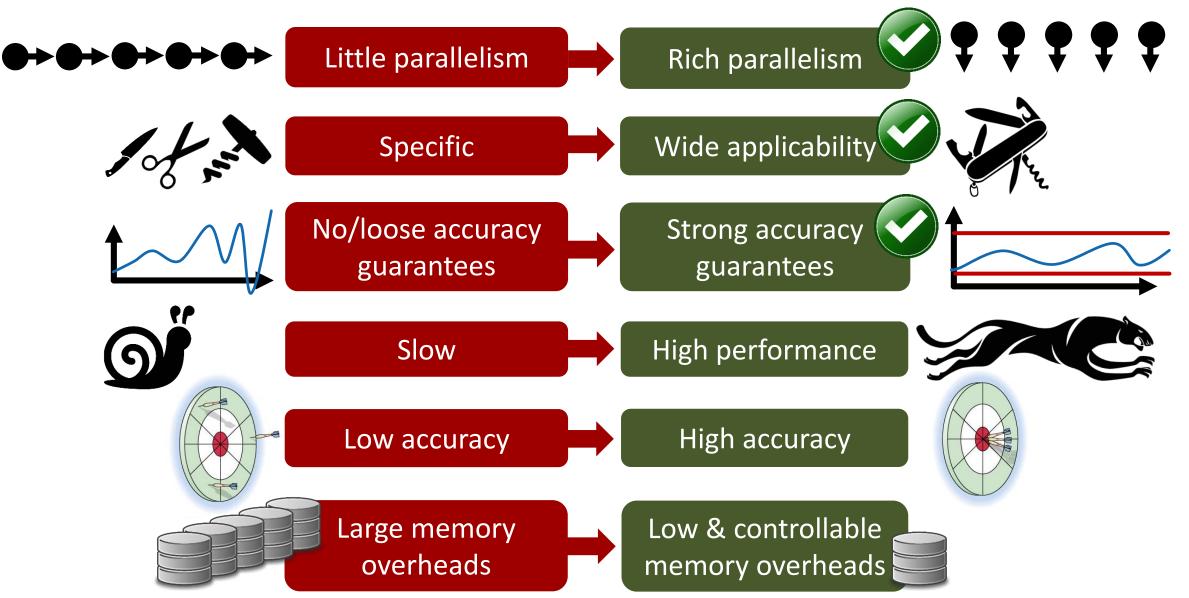


And Party States States

#### ... Many more data & a lot of strong theory results! $\left| P\left( \left| TC - \widehat{TC}_{1H} \right| \ge t \right) \le 2 \exp\left( -\frac{10 \ \kappa \ v}{\left( \sum_{v \in V} d(v)^2 \right)^2} \right) \right|$ Result Where Clas (work (depth) $E[(\widehat{|X|} - |X|)^2]$ (8) $\widehat{|X|}_{S}$ Eq. (1) BF $O(bd_n)$ $O(\log(bd_v))$ $X \cap Y$ **Theorem A.6.** Let $Y_1 =$ $O(kd_v)$ $O(\log d_v)$ $=E[(\widehat{|X|} - |X|)^2 |\mathcal{E}]P(\mathcal{E}) + E[(\widehat{|X|} - |X|)^2 |\neg \mathcal{E}]P(\neg \mathcal{E})$ (9)and assume we partition $\mathcal{X}_1, \cdots, \mathcal{X}_{\chi}$ such that estin independent. Then for any $|X \cap Y|$ PG (BF) PG (MH) $\leq (1+\varepsilon)E[(\widehat{|X|}-\kappa)^2|\mathcal{E}] + \frac{1+\varepsilon}{\varepsilon}E[(\kappa-|X|)^2|\mathcal{E}] + O(B_X^2\log^2 B_X) \cdot \exp(-B_X^{\Omega(1)})$ (10) $\left(\frac{ndB_X}{W}\right)$ $S = \sum_{i=1}^{n} C_i \frac{J}{1+J}$ O(ndk)Result $\operatorname{og}\left(\frac{B_X}{W}\right)$ $\leq \frac{(1+\varepsilon)B_X^2}{\iota^2} E[(\log(B_{X,0}/B_X) - \log(1-1/B_X)^{b|X|})^2 |\mathcal{E}] + O((\kappa - |X|)^2) + \exp(-B_X^{\Omega(1)})$ $O(\log k)$ $\widehat{|X|}_{S} \bigstar$ $P(|Y_1 - S| > t), P(|Y_k - S| > t)$ (11) $\frac{nd^2 \overset{`}{B}_X}{W}$ $\widehat{|X \cap Y|}_{AN}$ $O(nd^2k)$ $|\widehat{X} \cap \widehat{Y}|_{I}$ $\leq \frac{(1+\varepsilon)B_X^2}{^{12}}E[(\log(B_{X,0}/B_X) - \log(1-1/B_X)^{b|X|})^2|\mathcal{E}] + O(|X|/B_X)$ $\log d \log \left( \frac{B_X}{W} \right)$ $O\left(\log^2 k\right)$ (12) $|\widehat{X} \cap \widehat{Y}|_{h,l}$ $|\widehat{X \cap Y}|_{1H} \bigstar$ Eq. (7) 1-Hash $\leq \frac{(1+\varepsilon)^2 B_X^2}{L^2} e^{2b|X|/B_X} E[(B_{X,0}/B_X - (1-1/B_X)^{b|X|})^2 |\mathcal{E}] + O(|X|/B_X)$ (13)Constr. Memory (30)Reference time used $\leq \frac{(1+\varepsilon)^2 B_X^2}{L^2} e^{2b|X|/(B_X-1)} \cdot E[(B_{X,0}/B_X - (1-1/B_X)^{b|X|})^2]/P[\mathcal{E}] + O(|X|/B_X)$ (14)Doulion [46] $O(m) \quad O(pm)$ O(pm) $|X|_i[E(\widehat{|X|}_j) - |X|_j]$ (31) Colorful [47] O(m)Sketching [48] O(km) O(kn) $= \left( (1+\varepsilon)^2 + o(1) \right) \frac{B_X^2}{k^2} e^{2b|X|/(B_X-1)} \cdot E[(B_{X,0}/B_X - (1-1/B_X)^{b|X|})^2] + O(|X|/B_X)$ O(n+m)ASAP [49] (15)**H** GAP [50] $O(m)\dagger O(m')\dagger$ 🗳 Slim Gr. [51] $O(m) \quad O(pm)$ dif $|X|_i] \left| \left| [E(\widehat{|X|}_j) - |X|_j] \right|$ $= \left( (1+\varepsilon)^2 + o(1) \right) \frac{e^{2b|X|/(B_X-1)}}{h^2} Var[B_{X,0}] + O(|X|/B_X)$ $O\left(\frac{n}{TC^{1/3}}\right)$ Relative Eden et al. [52] (16)O(1)Assadi et al. [53] n/a (32) $\left(\frac{m^{1.41}}{TC^{0.82}}\right)$ $\leq \left( (1+\varepsilon)^2 + o(1) \right) e^{2b|X|/(B_X-1)} \cdot \left( e^{-\frac{b|X|}{B_X}} \frac{B_X}{b^2} - \frac{B_X}{b^2} - \frac{|X|}{b} \right) + O(|X|/B_X)$ Tětek [54] n/a (17)(35) $[]_j) - |X|_j]^2$ $\widehat{TC}_{AND}$ (BF) O(bm) O(n+m) $\bigcup \widehat{TC}_{kH}$ (MH) O(km) O(n+m) $\leq \left( (1+\varepsilon)^2 + o(1) \right) \left( e^{|X|b/(B_X-1)} \frac{B_X}{b^2} - B_X/b^2 - |X|/b \right) + O(|X|/B_X)$ (33) $\widehat{TC}_{1H}$ (MH) O(km) O(n+m)(36) (18) $\left(\frac{2\Delta}{b}\right)$ (34) $\leq \left( (1+\varepsilon)^2 + o(1) \right) \left( e^{|X|b/(B_X-1)} \frac{B_X}{b^2} - B_X/b^2 - |X|/b \right)$ (37) CSR (merge) (19)Work: $O(d_u + d_v)$ $\frac{2\Delta}{h}$ $P\left(\left|TC-\widetilde{T}\right|\right)$ (38) **Depth:** $O(\log(d_u + d$ $(v)^3$ Number of Threads 36

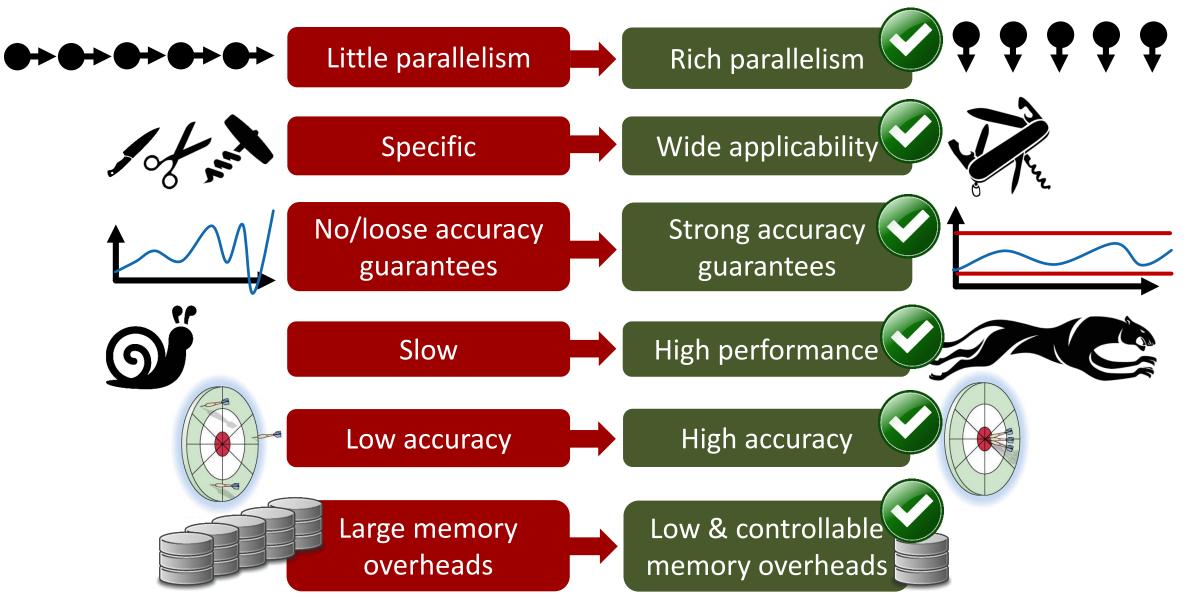


## **Approximate Graph Processing: Our Objectives**





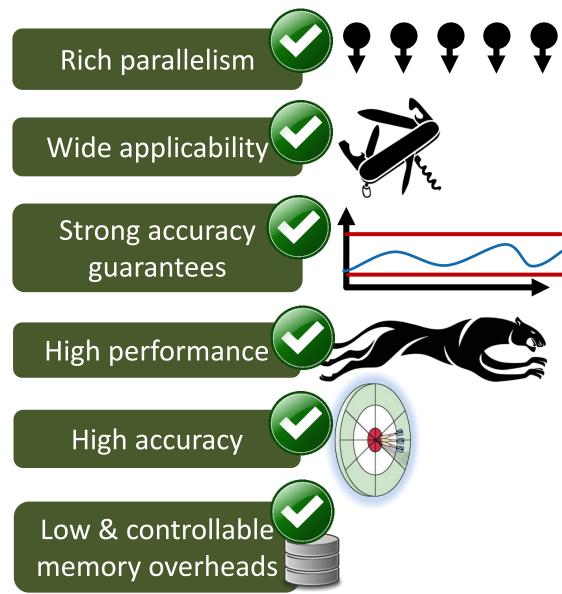
## **Approximate Graph Processing: Our Objectives**





**<u>Conclusion</u>**: ProbGraph Enables Approximate Graph Mining with...

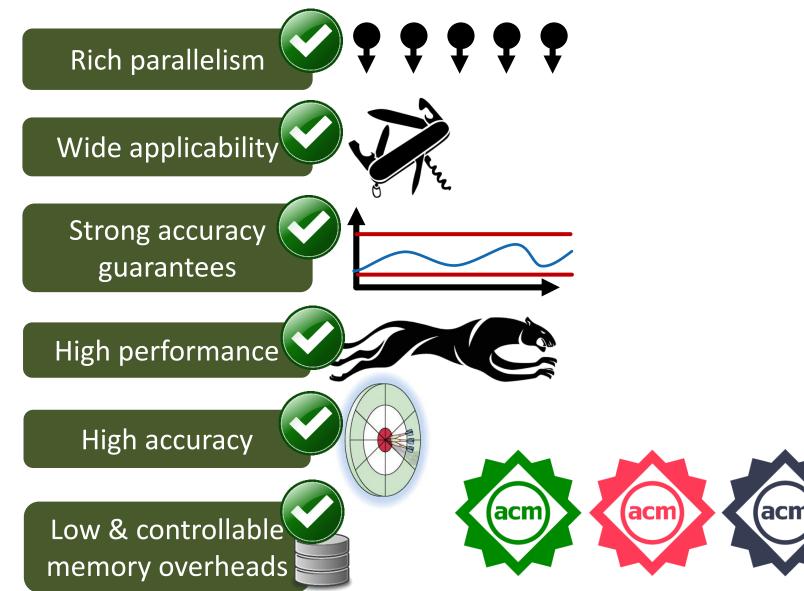
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**Conclusion:** ProbGraph Enables Approximate Graph Mining with...

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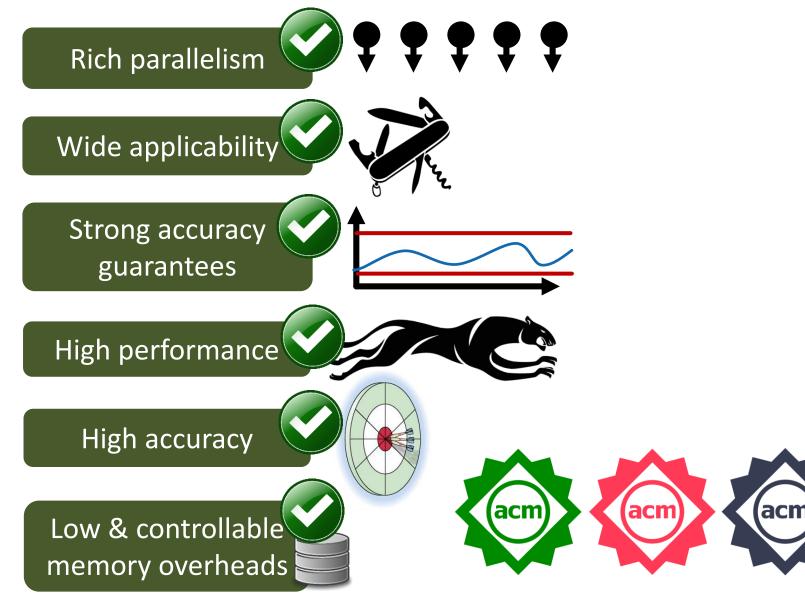




**<u>Conclusion</u>**: ProbGraph Enables Approximate Graph Mining with...

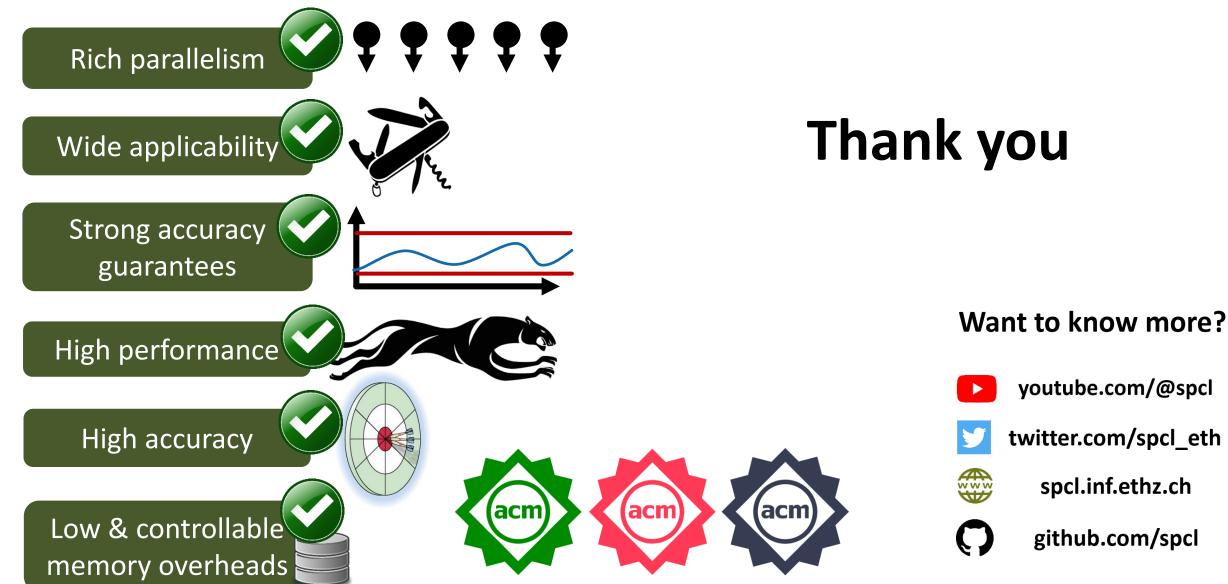
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Thank you





**<u>Conclusion</u>**: ProbGraph Enables Approximate Graph Mining with...



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**Conclusion:** ProbGraph Enables Approximate Graph Mining with...

