SlimFly: A Cost Effective Low-Diameter Network Topology

Maciej Besta, Torsten Hoefler
**DESIGNING AN EFFICIENT NETWORK TOPOLOGY**

- **Goals:**
  - Decrease network cost & power consumption
  - Preserve high bandwidth

- **How can the cost/power consumption be reduced?**
  
  *By lowering diameter!*

- **Intuition:** lower diameter means:
  - Fewer router buffers and thus SerDes (Serializers/Deserializers) traversed
    - reduces power consumption
  - Lower average path length
    - reduces the number of necessary cables and routers
DESIGNING AN EFFICIENT NETWORK TOPOLOGY

EXAMPLE: FULL-BANDWIDTH FAT TREE VS HOFFMAN-SINGLETON GRAPH

3-level fat tree:

Slim Fly based on the Hoffman-Singleton Graph [1]:

diameter = 4

diameter = 2
> ~50% fewer routers
> ~30% fewer cables

OVERVIEW OF OUR RESEARCH

Topology design
Optimizing towards Moore Bound

Attaching endpoints

Comparison of optimality

Cost, power, resilience analysis

Physical layout

Cost model

Cost & power results

Detailed case-study

Resilience

Routing and performance

Routing

Performance, latency, bandwidth
DESIGNING AN EFFICIENT NETWORK TOPOLOGY

TERMINOLOGY

- router radix
- network radix
- concentration
DESIGNING AN EFFICIENT NETWORK TOPOLOGY

GENERAL CONSTRUCTION SCHEME

- We establish a general construction approach with two phases:

**Connect routers:**
- select *diameter*
- select *network radix*
- maximize *number of routers*

**Attach endpoints**
- Derive *concentration* that provides full global bandwidth
DESIGNING AN EFFICIENT NETWORK TOPOLOGY
CONNECTING Routers

- Idea: optimize towards the Moore Bound (MB)
- Moore Bound [1]: upper bound on the number of routers in a graph with given diameter \( D \) and network radix \( k \).

\[
MB(D, k) = 1 + k \sum_{i=0}^{D-1} (k - 1)^i
\]

DESIGNING AN EFFICIENT NETWORK TOPOLOGY

CONNECTING ROUTERS: DIAMETER 2

- Example Slim Fly design for diameter = 2: MMS graphs [1]

Groups form a fully-connected bipartite graph
Designing an Efficient Network Topology

Connecting routers: Diameter 2

1. Select a prime power $q$
   \[ q = 4w + \delta; \]
   \[ w \in \mathbb{N} \quad \delta \in \{-1,0,1\}, \]
   A Slim Fly based on $q$
   Number of routers: $2q^2$
   Network radix: $(3q - \delta)/2$

2. Construct a finite field $\mathcal{F}_q$.
   Assuming $q$ is prime:
   \[ \mathcal{F}_q = \mathbb{Z}/q\mathbb{Z} = \{0,1, \ldots, q-1\} \]
   with modular arithmetic.

Example: $q = 5$

50 routers
network radix: 7

$\mathcal{F}_5 = \{0,1,2,3,4\}$
DESIGNING AN EFFICIENT NETWORK TOPOLOGY

CONNECTING ROUTERS: DIAMETER 2

1. Select a prime power $q$
   
   $q = 4w + \delta;$
   
   $w \in \mathbb{N}$  \hspace{0.5cm} $\delta \in \{-1, 0, 1\},$
   
   A Slim Fly based on $q$:
   
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DESIGNING AN EFFICIENT NETWORK TOPOLOGY
CONNECTING ROUTERS: DIAMETER 2

3 Label the routers

Set of routers:
\[ \{0,1\} \times \mathcal{F}_q \times \mathcal{F}_q \]

Example: \( q = 5 \)

Routers (0,..,)

(0,0,0) (0,0,1) (0,0,2) (0,0,3) (0,0,4)

(0,1,0) (0,1,1) (0,1,2) (0,1,3) (0,1,4)

(0,2,0) (0,2,1) (0,2,2) (0,2,3) (0,2,4)

(0,3,0) (0,3,1) (0,3,2) (0,3,3) (0,3,4)

(0,4,0) (0,4,1) (0,4,2) (0,4,3) (0,4,4)

Routers (1,..,)

(1,0,0) (1,0,1) (1,0,2) (1,0,3) (1,0,4)

(1,1,0) (1,1,1) (1,1,2) (1,1,3) (1,1,4)

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(1,4,0) (1,4,1) (1,4,2) (1,4,3) (1,4,4)

...
**Designing An Efficient Network Topology**

**Connecting routers: Diameter 2**

4. Find primitive element \( \xi \)

\[ \xi \in \mathbb{F}_q \] generates \( \mathbb{F}_q \):

All non-zero elements of \( \mathbb{F}_q \) can be written as \( \xi^i; \ i \in \mathbb{N} \)

5. Build Generator Sets

\[ X = \{1, \xi^2, ..., \xi^{q-3}\} \]

\[ X' = \{\xi, \xi^3, ..., \xi^{q-2}\} \]

Example: \( q = 5 \)

\[ \mathbb{F}_5 = \{0, 1, 2, 3, 4\} \]

\[ \xi = 2 \]

\[ 1 = \xi^4 \mod 5 = 2^4 \mod 5 = 16 \mod 5 \]

\[ X = \{1, 4\} \]

\[ X' = \{2, 3\} \]
Designing an Efficient Network Topology

Connecting routers: Diameter 2

6 Intra-group connections

Two routers in one group are connected iff their “vertical Manhattan distance” is an element from:

\[ X = \{1, \xi^2, \ldots, \xi^{q-3}\} \quad \text{(for subgraph 0)} \]
\[ X' = \{\xi, \xi^3, \ldots, \xi^{q-2}\} \quad \text{(for subgraph 1)} \]

Example: \( q = 5 \)

Take Routers \((0,0,\ldots)\)

\[ X = \{1, 4\} \]
**DESIGNING AN EFFICIENT NETWORK TOPOLOGY**

**CONNECTING ROUTERS: DIAMETER 2**

### Intra-group connections

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### Example: \( q = 5 \)

Take Routers \((0,0, \_)\)

\[ X = \{1, 4\} \]

---

**Diagram:**

- Intra-group connections are represented by lines between routers in the same group.
- Example subgraphs are shown with highlighted connections.
- 

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DESIGNING AN EFFICIENT NETWORK TOPOLOGY

CONNECTING ROUTERS: DIAMETER 2

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E Example: \( q = 5 \)

Take Routers \((1,4,.)\)

\[ X' = \{2,3\} \]
**DESIGNING AN EFFICIENT NETWORK TOPOLOGY**

**CONNECTING ROUTERS: DIAMETER 2**

7 Inter-group connections

Router \((0, x, y) \leftrightarrow (1, m, c)\)

iff \(y = mx + c\)

Example: \(q = 5\)

- Take Router \((1,0,0)\)
  - \((1,0,0) \leftrightarrow (0, x, 0)\)
  - \(m = 0, c = 0\)

- Take Router \((1,1,0)\)
  - \((1,0,0) \leftrightarrow (0, x, x)\)
  - \(m = 1, c = 0\)
DESIGNING AN EFFICIENT NETWORK TOPOLOGY

CONNECTING ROUTERS: DIAMETER 2

- Viable set of configurations
  - 10 SF networks with *the number of endpoints* < 11,000 (compared to 6 balanced Dragonflies [1])
- Let’s pick *network radix = 7*...
  - ... We get the Hoffman-Singleton graph (attains the Moore Bound)
DESIGNING AN EFFICIENT NETWORK TOPOLOGY

ATTACHING ENDPOINTS: DIAMETER 2

- How many endpoints do we attach to each router?
- As many to ensure *full global bandwidth*:
  - Global bandwidth: the theoretical cumulative throughput if all endpoints simultaneously communicate with all other endpoints in a steady state

\[
\text{concentration} = ?
\]
DESIGNING AN EFFICIENT NETWORK TOPOLOGY

ATTACHING ENDPOINTS: DIAMETER 2

1. Get load \( l \) per router-router channel (average number of routes per channel)

\[
l = \frac{\text{total number of routes}}{\text{total number of channels}}
\]

2. Make the network balanced, i.e.,:

- each endpoint can inject at full capacity
- local uplink load = number of endpoints = \( l \)

network radix = 67% of router radix

concentration = 33% of router radix
**Comparison to Optimality**

- How close is the presented Slim Fly network to the Moore Bound?

![Graph showing network performance](image)

*Each number shows the fraction of the upper bound of a data point (higher is better).*

- Networks with diameter = 2:
  - 88%
  - 21%
  - 1.6%
  - 1.2%

**Topology**
- Moore Bound 2
- Slim Fly MMS
- Flat. Butterfly
- Long Hop
- Fat tree
**OVERVIEW OF OUR RESEARCH**

- **Topology design**
  - Optimizing towards Moore Bound

- **Cost, power, resilience analysis**
  - Physical layout
  - Cost model
  - Comparison targets

- **Routing and performance**
  - Routing
  - Performance, latency, bandwidth

- **Attaching endpoints**
  - Comparison of optimality

- **Routing and performance**

**Comparison targets**
- Structure analysis
- Cost model
- Cost & power results
- Detailed case-study
- Resilience
Mix (pairwise) groups with different cabling patterns to shorten inter-group cables.
PHYSICAL LAYOUT
PHYSICAL LAYOUT
Merge groups pairwise to create racks
PHYSICAL LAYOUT
Racks form a fully-connected graph
**PHYSICAL LAYOUT**

**SlimFly:**
- ~50% fewer intra-group cables
- \(2(q-1)\) inter-group cable between two groups
- \(~25\%\) fewer routers

**Dragonfly:**
- ~33% higher endpoint density
- One inter-group cable between two groups

~25% fewer routers
**COST COMPARISON**

**COST MODEL**

- Electric cables, avg length: 1m
- Top-of-rack routers
- 1 meter of overhead for each local link
- Optic cables, length: Manhattan distance
- 2m of overhead for each global link
- Racks arranged as close to a square as possible

*Most cables skipped for clarity*
COST COMPARISON
CABLE COST MODEL

- Cable cost as a function of distance
  - The functions obtained using linear regression*
  - Cables used:
    Mellanox IB FDR10 40Gb/s QSFP

- Other used cables:
  - Mellanox IB QDR 56Gb/s QSFP
  - Mellanox Ethernet 40Gb/s QSFP
  - Mellanox Ethernet 10Gb/s SFP+
  - Elpeus Ethernet 10Gb/s SFP+

*Prices based on:

\[
f(x) = 0.4079x + 0.5771
\]

\[
f(x) = 0.0919x + 2.7452
\]
COST COMPARISON

ROUTER COST MODEL

- Router cost as a function of radix
  - The function obtained using linear regression*
  - Routers used:

  Mellanox IB FDR10

  Mellanox Ethernet 10/40 Gb

*Prices based on:
COMPARISON TARGETS
LOW-RADIX TOPOLOGIES

Torus 3D

Cray XE6

IBM BG/Q

Torus 5D

Hypercube

NASAs Pleiades

Infinetics

Long Hop [1]

**COMPARISON TARGETS**

**HIGH-RADIX TOPOLOGIES**

- **Fat tree [1]**
- **Dragonfly [3]**
- **Random Topologies [4,5]**
- **Flattened Butterfly [2]**

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COST COMPARISON

RESULTS

![Graph showing cost comparison across different topologies]
A Slim Fly with:
- $N = 10,830$
- $k = 43$
- $N_r = 722$
# Cost & Power Comparison

## Detailed Case-Study: High-Radix Topologies

<table>
<thead>
<tr>
<th>Topology</th>
<th>Fat tree</th>
<th>Random</th>
<th>Flat. Butterfly</th>
<th>Dragonfly</th>
<th>Slim Fly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endpoints ((N))</td>
<td>19,876</td>
<td>40,200</td>
<td>20,736</td>
<td>58,806</td>
<td><strong>10,830</strong></td>
</tr>
<tr>
<td>Routers ((N_r))</td>
<td>2,311</td>
<td>4,020</td>
<td>1,728</td>
<td>5,346</td>
<td><strong>722</strong></td>
</tr>
<tr>
<td>Radix ((k))</td>
<td><strong>43</strong></td>
<td><strong>43</strong></td>
<td><strong>43</strong></td>
<td><strong>43</strong></td>
<td><strong>43</strong></td>
</tr>
<tr>
<td>Electric cables</td>
<td>19,414</td>
<td>32,488</td>
<td>9,504</td>
<td>56,133</td>
<td><strong>6,669</strong></td>
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<tr>
<td>Fiber cables</td>
<td>40,215</td>
<td>33,842</td>
<td>20,736</td>
<td>29,524</td>
<td><strong>6,869</strong></td>
</tr>
</tbody>
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<tbody>
<tr>
<td>Endpoints ((N))</td>
<td><strong>10,718</strong></td>
<td><strong>9,702</strong></td>
<td><strong>10,000</strong></td>
<td><strong>9,702</strong></td>
<td><strong>10,830</strong></td>
</tr>
<tr>
<td>Routers ((N_r))</td>
<td>1,531</td>
<td>1,386</td>
<td>1,000</td>
<td>1,386</td>
<td><strong>722</strong></td>
</tr>
<tr>
<td>Radix ((k))</td>
<td>35</td>
<td>28</td>
<td>33</td>
<td>27</td>
<td><strong>43</strong></td>
</tr>
<tr>
<td>Electric cables</td>
<td>7,350</td>
<td>6,837</td>
<td>4,500</td>
<td>9,009</td>
<td><strong>6,669</strong></td>
</tr>
<tr>
<td>Fiber cables</td>
<td>24,806</td>
<td>7,716</td>
<td>10,000</td>
<td>4,900</td>
<td><strong>6,869</strong></td>
</tr>
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<tbody>
<tr>
<td>Cost per node ([$])</td>
<td>2,346</td>
<td>1,743</td>
<td>1,570</td>
<td>1,438</td>
<td><strong>1,033</strong></td>
</tr>
<tr>
<td>Power per node ([\text{W}])</td>
<td>14.0</td>
<td>12.04</td>
<td>10.8</td>
<td>10.9</td>
<td><strong>8.02</strong></td>
</tr>
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**STRUCTURE ANALYSIS**

**RESILIENCY**

- Disconnection metrics*
- Other studied metrics:
  - Average path length (increase by 2); SF is 10% more resilient than DF

<table>
<thead>
<tr>
<th>$\approx N$</th>
<th>Torus3D</th>
<th>Torus5D</th>
<th>Hypercube</th>
<th>Long Hop</th>
<th>Fat tree</th>
<th>Dragonfly</th>
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<th>Random</th>
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<tr>
<td>512</td>
<td>30%</td>
<td>-</td>
<td>40%</td>
<td>55%</td>
<td>35%</td>
<td>-</td>
<td>55%</td>
<td>60%</td>
<td>60%</td>
</tr>
<tr>
<td>1024</td>
<td>25%</td>
<td>40%</td>
<td>40%</td>
<td>55%</td>
<td>40%</td>
<td>50%</td>
<td>60%</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2048</td>
<td>20%</td>
<td>-</td>
<td>40%</td>
<td>55%</td>
<td>40%</td>
<td>55%</td>
<td>65%</td>
<td>65%</td>
<td>65%</td>
</tr>
<tr>
<td>4096</td>
<td>15%</td>
<td>-</td>
<td>45%</td>
<td>55%</td>
<td>55%</td>
<td>60%</td>
<td>70%</td>
<td>70%</td>
<td>70%</td>
</tr>
<tr>
<td>8192</td>
<td>10%</td>
<td>35%</td>
<td>45%</td>
<td>55%</td>
<td>60%</td>
<td>65%</td>
<td>-</td>
<td>75%</td>
<td>75%</td>
</tr>
</tbody>
</table>

*Missing values indicate the inadequacy of a balanced topology variant for a given $N$
OVERVIEW OF OUR RESEARCH

Topology design
- Optimizing towards Moore Bound

Attaching endpoints
- Comparison of optimality

Cost, power, resilience analysis
- Physical layout
- Cost model
- Comparison targets
- Cost & power results
- Detailed case-study

Routing and performance

Routing
- Performance & routing
- Optimizing towards Moore Bound
- Attaching endpoints
- Comparison of optimality

Cost model
- Performance, latency, bandwidth
- Resilience
**PERFORMANCE & ROUTING**

- Cycle-accurate simulations [1]
- Routing protocols:
  - Minimum static routing
  - Valiant routing [2]
  - Universal Globally-Adaptive Load-Balancing routing [3]
    - **UGAL-L**: each router has access to its local output queues
    - **UGAL-G**: each router has access to the sizes of all router queues in the network

---

**PERFORMANCE & ROUTING**

**MINIMUM ROUTING**

1. **Intra-group connections**
   - Path of length 1 or 2 between two routers

2. **Inter-group connections (different types of groups)**
   - Path of length 1 or 2 between two routers

3. **Inter-group connections (identical types of groups)**
   - Path of length 2 between two routers
PERFORMANCE & ROUTING
RANDOM UNIFORM TRAFFIC

Routing protocol
- Slim Fly (Valiant)
- Slim Fly (UGAL–G)
- Slim Fly (Minimum)
- Dragonfly (UGAL–L)
- Slim Fly (UGAL–L)
- Fat Tree (ANCA)

[Graph showing latency vs. offered load with different routing protocols]

Offered load
0.00 0.25 0.50 0.75 1.00
Latency [cycles]
50 40 30 20 10
OTHER
RESULTS

- Bisection b.
- Avg. distance
- Oversubscription analysis

Bit reverse
- Bit complement
- Shuffle
- Shift
- Adversarial

Buffer size analysis
- Other cost & power results

Oversubscription analysis

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<td>6,885</td>
<td>6,669</td>
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<td>1,012</td>
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<td>Cost per node ($)</td>
<td>1,365</td>
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CONCLUSIONS

Topology design

Optimizing towards the Moore Bound reduces expensive network resources

Advantages of SlimFly

Cost & power | Resilience | Performance | Diameter | Avg. distance | Bandwidth
---|---|---|---|---|---

Optimization approach

Combining mathematical optimization and current technology trends effectively tackles challenges in networking
A LOWEST-DIAMETER TOPOLOGY
→ Viable set of configurations
→ Resilient

A COST & POWER EFFECTIVE TOPOLOGY
→ 25% less expensive than Dragonfly,
→ 26% less power-hungry than Dragonfly

A HIGH-PERFORMANCE TOPOLOGY
→ Lowest latency
→ Full global bandwidth

http://spcl.inf.ethz.ch/Research/Scalable_Networking/SlimFly

Thank you for your attention
**DEADLOCK FREEDOM**

**MINIMUM STATIC ROUTING**

- Assign two virtual channels (VC0 and VC1) to each link
- For a 1-hop path use VC0
- For a 2-hop path use VC0 (hop 1) and VC1 (hop 2)
- One can also use the DFSSSP scheme [1]

DEADLOCK FREEDOM

ADAPTIVE ROUTING

- Simple generalization of the previous scheme
- Assign four virtual channels (VC0 – VC3) to each link
- For hop $k$ path use $VC_k$, $0 \leq k \leq 3$
**PERFORMANCE**

- Bit permutation traffic

![Diagram](image-url)
PERFORMANCE

- Shift traffic

$$d = \left( s \mod \frac{N}{2} \right) + \frac{N}{2}$$

$$d = s \mod \frac{N}{2}$$
PERFORMANCE

- Worst-case traffic
**PERFORMANCE**

- Buffer sizes (UGAL-L, worst-case traffic)
- Oversubscription (64 flits)
POWER COMPARISON

POWER MODEL

- Model similar to [1],
  - Each router port has four lanes,
  - Each lane has one SerDes,
  - Each SerDes consumes 0.7 W
  - Other parameters as in the cost model

---

COST & POWER COMPARISON

DETAILED CASE-STUDY: HIGH-RADIX TOPOLOGIES

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STRUCTURE ANALYSIS

AVERAGE DISTANCE

Random uniform traffic using minimum path routing

- Torus 3D
- Hypercube
- Torus 5D
- Long Hop
- Fat Tree
- Flat. Butterfly
- Random top.
- Dragonfly
- Slim Fly

Average number of hops vs Network size [endpoints]
STRUCTURE ANALYSIS
BISECTION BANDWIDTH (BB)

*BB approximated with the Metis partitioner [1]

[1] G. Karypis, V. Kumar. A fast and high quality multilevel scheme for partitioning irregular graphs. ICPP’95
**DESIGNING AN EFFICIENT NETWORK TOPOLOGY**

**CONNECTING ROUTERS: DIAMETER 2**

*Intra-group connections*

- Router \((0, x, y) \leftrightarrow (0, x, y')\) \(\text{iff} \ y - y' \in X\)
- Router \((0, m, c) \leftrightarrow (0, m, c')\) \(\text{iff} \ c - c' \in X'\)

*Example: \(q = 5\)*

Take Routers \((0,0,\ldots)\)

- \((0,0,0), (0,0,1): y - y' = 1 \in X \checkmark\)
- \((0,0,0), (0,0,2): y - y' = 2 \notin X \times\)
- \((0,0,1), (0,0,2): y - y' = 1 \in X \checkmark\)
- \((0,0,0), (0,0,4): y - y' = 4 \in X \checkmark\)

\(X = \{1,4\}\)
**Designing an Efficient Network Topology**

**Connecting Routers: Diameter 2**

6. *Intra-group connections*

Router \((0, x, y) \leftrightarrow (0, x, y')\)

iff \(y - y' \in X\)

Router \((0, m, c) \leftrightarrow (0, m, c')\)

iff \(c - c' \in X'\)

**Example: \(q = 5\)**

Take Routers \((1,4,.)\)

\(X' = \{2,3\}\)

- \((1,4,0), (1,4,1): y - y' = 1 \notin X'\) (X)
- \((0,0,0), (0,0,2): y - y' = 2 \in X'\) (✓)
- \((0,0,1), (0,0,4): y - y' = 3 \in X'\) (✓)
- \((0,0,0), (0,0,4): y - y' = 4 \notin X'\) (X)

...
**DESIGNING AN EFFICIENT NETWORK TOPOLOGY**

**CONNECTING ROUTERS: DIAMETER 2**

**7 Inter-group connections**

Router $(0, x, y) \leftrightarrow (1, m, c)$

iff $y = mx + c$

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**Example: $q = 5$**

Take Router $(1,1,0)$

- $(0,0,0): y = 0 \quad mx + c = 0 \; \checkmark$
- $(0,1,1): y = 1 \quad mx + c = 1 \; \checkmark$
- $(0,2,2): y = 2 \quad mx + c = 2 \; \checkmark$
- ...  
- $(0,4,4): y = 4 \quad mx + c = 4 \; \checkmark$

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Diagram showing connections between routers $(0,0,0)$, $(0,1,1)$, $(0,2,2)$, $(0,3,3)$, $(0,4,4)$, and $(1,1,0)$.