

# LogP - A Model for small Messages in InfiniBand

Torsten Hoefler, Torsten Mehlan, Frank Mietke, and Wolfgang Rehm

Chemnitz University of Technology  
Dept. of Computer Science  
Chemnitz, 09107 GERMANY  
{htor, tome, mief, rehm}@cs.tu-chemnitz.de

## Abstract

*Accurate models of parallel computation are often crucial to optimize parallel algorithms for their running time. In general the easier the model's use and the smaller the number of parameters and interdependencies among them, the more inaccuracies are introduced by simplification. On the other hand a too complex model is unusable. We show that it is possible to derive a relatively accurate and easy model for small message performance over the InfiniBand network. This model allows the developer to gain knowledge about the inherent parallelism of a specific InfiniBand hardware and encourages him to use this parallelism efficiently. Several well known models hide this feature and some of them even penalize the use of parallelism because the model designers were not aware of new emerging architectures like InfiniBand.*

## 1 Introduction

Communication models play an important role in designing and optimizing parallel algorithms or applications. A model assists the programmer in understanding all important aspects of the underlying architecture without knowing unnecessary details. This abstraction is useful to simplify the algorithm design and to enable the use of mathematical proves and runtime assessments of algorithms. Most models enable non-computer scientists to understand everything they need for programming and computer architects to give running time estimations for different architectures. These models have to be very accurate and should reflect all important underlying hardware properties. But an architectural or communication model must also be feasible for programmers. This means that the number of parameters and the model functions must not be

too complex. The programmer has to understand the model and all its implications. It is easy to see that the accuracy and the ease of use are conflicting and the designer of a network model has to find the golden mean.

### 1.1 Related Work

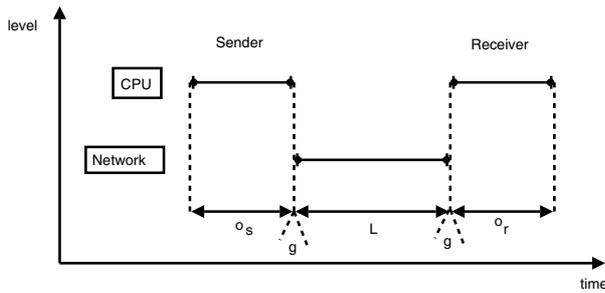
Many different models have been developed in the past. There are models for specific network architectures [17, 3] or for the shared memory paradigm such as CICO [16, 7]. Other models like PRAM [6, 14], BSP [21],  $C^3$  [9] or LogP [4] aim to be architecture independent and to give a general estimation of programming parallel systems. These are quite inaccurate due to their high level of abstraction. Several comparative studies [18, 8, 2, 11] are available for assessing the accuracy of subsets of these models. Our comparative study [11] and the prediction of the MPI\_BARRIER latency [10] with LogP shows that the LogP model is quite accurate for small messages. Many efforts [1, 19, 13, 15] have been made to enhance the model in its accuracy for different network architectures and large messages.

## 2 The LogP Model

Several studies [2, 10, 11] have shown that the LogP model is very accurate for small messages. This accuracy, and the simplicity of this model drive to the decision to base further developments on it. The LogP model will be explained shortly in the following. It reflects all important aspects of the communication behavior of parallel systems which are seen as a collection of loosely coupled computers. Each computer has one or more processors with main memory, works asynchronously and is equipped with a network interconnect to reach all other computers. Basically, the model is based on four parameters:

- $L$  - communication delay (**upper** bound to the latency for NIC-to-NIC messages from one processor to another)
- $o$  - communication overhead (time that a processor is engaged in transmission or reception of a single message, split up into  $o_s$  for send overhead and  $o_r$  for receive overhead)
- $g$  - gap (indirect communication bandwidth, minimum interval between consecutive messages,  $\text{bandwidth} \sim \frac{1}{g}$ )
- $P$  - number of processors

The parameters of the LogP model can be divided into two layers, the CPU-Layer and the Network-Layer. The  $o$ -parameter can also be subdivided into one parameter on the receiver side ( $o_r$ ) and another one on the sender side ( $o_s$ ). The according visualization of the different parameters for a LogP compliant network (e.g. Ethernet) can be seen in figure 1.



**Figure 1. Visualization of the LogP parameters**

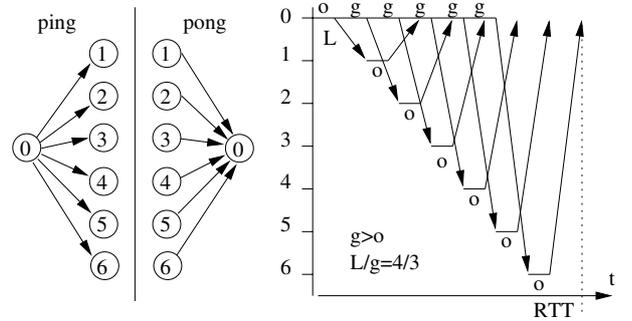
The parameters have to adhere to several assumptions to make the model fully functional:

- $\left\lceil \frac{L}{g} \right\rceil$  - count of messages that can be in transmission on the network from one to any other processor in parallel (network capacity)
- $L$ ,  $o$  and  $g$  are measured as multiples of the processor cycle

An additional study [5] describes options of assessing the network parameters for real-life supercomputers. This can be very helpful to gain a deeper knowledge about the model's characteristics.

### 3 InfiniBand Benchmarks

This section compares different InfiniBand benchmark curves with the LogP prediction. The Benchmark determines  $1 : P - P : 1$  Round Trip Times ( $RTT$ ) and CPU send overheads ( $o$ ) for posting the send request. We examine only RMDA-Write because it is the fastest way to transmit data over InfiniBand, and it exhibits no receive overhead  $o_r$ . Thus the only remaining overhead  $o_s$  is called  $o$  in the remainder of this paper. All curves are normalized to the number of addressed hosts. Thus, the  $RTT$  costs or  $o$  costs per message, called  $RTT/P$  and  $o/P$  are displayed over the number of addressed hosts. The LogP model predicts a constant  $o$  and the following  $RTT$  needed for transmitting a single packet to  $P$  hosts and back from all receivers to the sender. The LogP communication di-



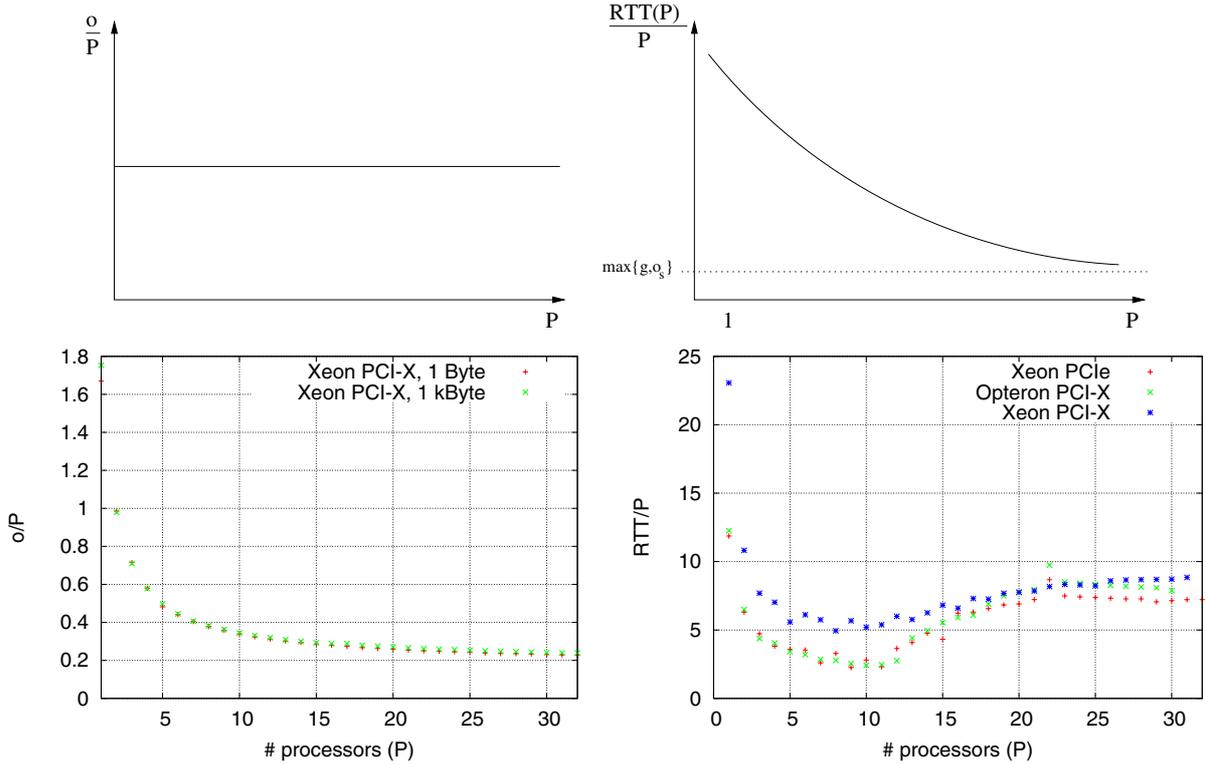
**Figure 2.  $1 : P - P : 1$  Ping Pong Benchmark Scheme and LogP Communication Structure**

agram is shown in figure 2 and the derived LogP  $RTT$  predictions are as follows:

$$\begin{aligned} RTT &= o + (P - 1) \cdot \max\{o, g\} + L + o + L \\ &= 2L + 2o + (P - 1) \cdot \max\{o, g\} \end{aligned} \quad (1)$$

The expected graph signature of the  $RTT/P$  (cp. equation (1)) normalized by  $P$  is shown in figure 3.

The practical InfiniBand  $1 : P - P : 1$   $o$  benchmark result is shown in the lower left of figure 3 for 1 Byte and 1024KB. It can be assumed that there is no difference, and the overhead does not depend on the size of the posted message. The LogP prediction for  $o$  is constant and cannot express the benchmarked function signature properly. The  $RTT/P$  benchmark results for 1 byte messages are shown in the upper right of figure 3. The small message function has a totally different signature than the LogP prediction and the time per message has a global minimum at  $P \approx 10$ . This special behavior of InfiniBand<sup>TM</sup> has also been investigated in



**Figure 3.**  $o(P)$  and  $1 : P - P : 1$   $RTT(P)$  **LogP predictions (up) and Benchmark Results (down)**

“A Communication Model for Small Messages with InfiniBand” [12] and is briefly analyzed in the following section.

## 4 The LoP Model

The original LoP model was presented in “A Communication Model for Small Messages with InfiniBand” [12]. The model investigates the phenomenon of the local minimum of the cost per message measured in section 3 and the varying overhead  $o$ . We derived very accurate model functions which describe the exact behavior of the InfiniBand hardware. The performance assessment function is represented by a parametrized model function. The values of the real parameters  $\lambda_1 \dots \lambda_6$  are derived by fitting the unparametrized model function to the benchmark results. Thus, not all  $\lambda_n$  parameters have a meaning in the “real world” and have to be seen as mathematical constructs to fit the benchmark function efficiently.  $P$  equals to the number of addressed hosts.

### 4.1 Overhead Model

The overhead can be modelled as a simple pipeline start up function, due to several cache effects:

$$o(P) = \frac{\lambda_1}{\lambda_2 + P}$$

The function signature is shown on the left side of figure 4.

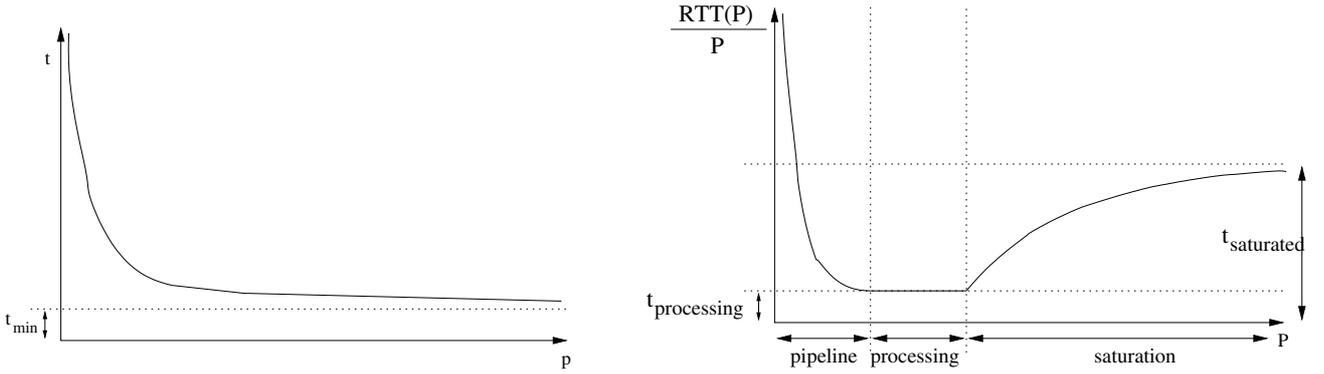
### 4.2 RTT Model

The model for the  $RTT$  is depicted on the right side of figure 4 and is divided into three sections: The first section can be described with a typical pipeline start up function due to several cache effects and the  $o(P)$  function:

$$t_{pipeline} = \frac{\lambda_1}{\lambda_2 + P}$$

The second section is defined by the maximum  $CPU \rightarrow NIC \rightarrow NIC \rightarrow CPU$  throughput or packet processing rate of the NIC, and is thus defined as constant:

$$t_{processing} = \lambda_3$$



**Figure 4.  $o$  (left) and  $RTT$  (right) Model**

The third section reflects the network saturation which typically behaves like an exponential function as:

$$t_{saturation} = \lambda_4 \cdot (1 - e^{\lambda_5 \cdot (P - \lambda_6)})$$

$\lambda_4$  and  $\lambda_5$  influence the shape of the function and  $\lambda_6$  introduces a  $P$ -offset. Altogether the  $RTT$  can be described with the following abstract model, which is depicted on the right side of figure 4:

$$\begin{aligned} RTT(P)_{\lambda_1 \dots \lambda_6} &= t_{pipeline} + t_{processing} + t_{saturation} \\ &= \frac{\lambda_1}{\lambda_2 + p} + \lambda_3 + \lambda_4 \cdot (1 - e^{\lambda_5 \cdot (p - \lambda_6)}) \end{aligned}$$

It is nearly impossible to handle six parameters as well as an exponential function to design optimal algorithms and even the mathematical proves are very hard to do. Another disadvantage is the complexity of finding the optimal parameters for a given set of benchmarks (we used a direct search in a six dimensional space with the Nelder Mead simplex algorithm [20]).

Thus, this model is practically unusable. Basing on the relatively simple LogP model and the fact that it covers most of our architectural needs, we decided to enhance it but keep its simple linear nature. The derivation of the new simplified LoP model is called LogfP because the additional parameter  $f$  defines the maximal number of send operations where no  $g$  is needed.

## 5 LogfP - A simplified LoP Model

The simplified LoP model, named LogfP, is derived from the original LogP model. The main characteristics and the ease of use are retained in the new design. Figure 5 shows the benchmarked  $o/P$  and  $RTT/P$  values, our proposed model function and the LogP pre-

dicted function. The  $f$  parameter indicates the number of messages where no  $g$  has to be accounted, which are essentially for  $f$  free.

### 5.1 Overhead Model

The overhead model is simply the pipeline function stated in 4.1. The parameters are more mnemonic:

$$o(P) = o_{min} + \frac{o_{max}}{P} \quad (2)$$

Where  $o_{min}$  is the lowest achievable  $o(P)$  for  $P \rightarrow \infty$  which is  $0.18\mu s$  in our benchmark in figure 3. The maximal value for  $o(P)$ ,  $o_{max}$  is exactly  $o(1)$  and  $1.6\mu s$  in our example. It defines the shape of the prediction function. Both parameters are very easy to derive if you have just two measurement results,  $o(1)$  and  $o(\infty) \approx o(x)$  for a sufficiently huge  $x$  (e.g.  $x = 1000$ ). This model is still extremely accurate and easy to use. The model's prediction and the measured values are shown in the left of figure 5. The  $o(P)$  parameter is very important to assess the CPU load for each send operation. It does not play a big role for the send process itself because the  $L$  parameter is usually 10 to 100 times bigger for InfiniBand. Thus, the complicated  $o(P)$  could be replaced with the scalar  $o$  from the LogP model for network transmissions (this introduces a slight inaccuracy but reduces the number of parameters by one!).

### 5.2 RTT Model

Our model is nearly the same as the LogP model. The difference is that the  $g$  parameter is not paid for every message after the first one. We assume that multiple small messages can be processed simultaneously in the network hardware (cmp. [12]) and sent simultaneously across the network. Thus,  $g$  is only paid for

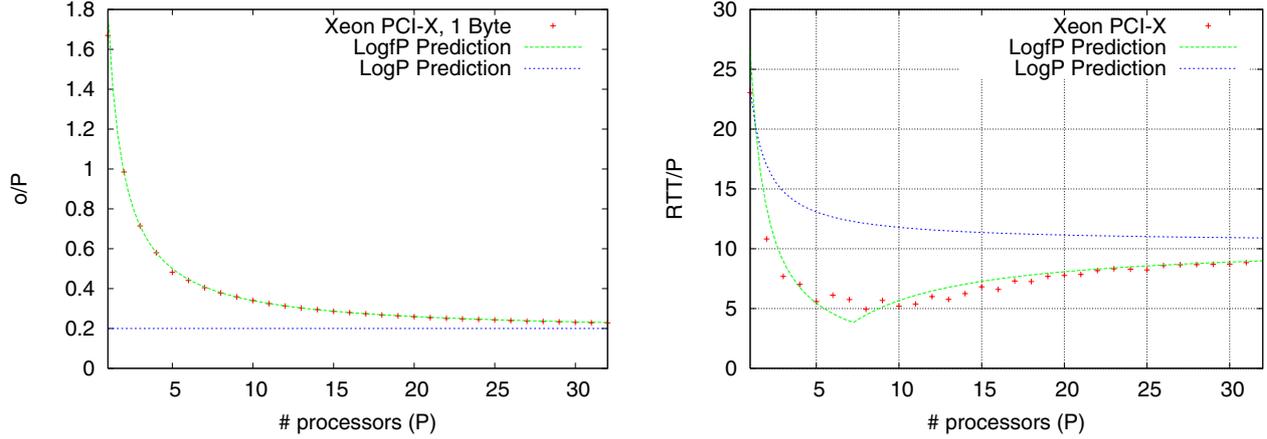


Figure 5.1 :  $P - P : 1 o$  and  $RTT$  Benchmark Results with Predictions

every message after  $f$  messages have been sent, which means that the first  $f$  small messages are essentially for free in our model. It is obvious that this cannot hold for large messages, due to the limited bandwidth. Thus, the Round Trip Time can be modelled as:

$$\begin{aligned} \forall (P \leq f) \quad RTT(P) &= 2L + P \cdot o_s(P) + o_s(1) \\ \forall (P > f) \quad RTT(P) &= 2L + o(P) + o_s(1) + \\ &\quad \max\{(P-1) \cdot o(P), (P-f) \cdot g\} \end{aligned}$$

It is easy to see that our simple modification of introducing the  $f$  parameter enhances the accuracy of the model significantly. The LogfP model is quite accurate for the prediction of small messages while the LogP model overestimates all  $RTT$ s. The introduction of the  $o_{min,max}$  parameters enhances the  $o$  modelling of the LogP model. LogP underestimates the needed CPU time to send a message due to its constant nature.

### 5.3 LogfP Parameter Assessment

All LogfP parameters can be gathered from the  $1 : P - P : 1$  benchmark described above. They are explained in the following:

- $o_{min}$  - equals to  $o(\infty)/P$  of the  $o(P)$  benchmark
- $o_{max}$  - equals to  $o(1)/P$  of the  $o(P)$  benchmark
- $L$  - equals  $\frac{RTT(1)-2o_{min}-2o_{max}}{2P}$  of the  $RTT(P)$  benchmark
- $g$  - equals  $RTT(\infty)/P$  of the  $RTT(P)$  benchmark
- $f$  - is the global minimum of the  $RTT(P)/P$  curve

- $P$  - number of processors

These parameters can easily be measured and used for modelling the running time of parallel algorithms which use small messages (e.g. the MPI\_BARRIER algorithm).

## 6 Conclusions and Future Work

We have shown that simple modifications can enhance the accuracy of the LogP model significantly. These modifications are also applicable to a developer's daily task to optimize algorithms. The predictions of the LoP model are much more accurate but due to their complexity not usable. Our model encourages the programmer to use the inherent hardware parallelism in the transmission of small messages, because sending the first  $f$  messages is for free (despite  $o$ ). Our model should be more accurate than the LogP model for other hardware offloading based networks where most packet processing is done in hardware (e.g. Quadrics, Myrinet). Future work includes enhancing the model also for large messages (cmp. LogGP [1]) and to provide several use-cases of the model to the community.

### 6.1 Acknowledgments

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